Reading: Bahnemann.Chapter5 Bahnemann_Ex5-4 (Problem 1)
Model: Source Text
Problem Type: Estimate the excess severity behaviour and determine the underlying distribution.

Given Grouped sample data for 1000 policies

Group	# Claims	Total Loss	Severity	$F_{1000}(x)$	$E_{1000}[\hat{X};x]$	$e_{1000}(x)$
0 - 100	100	6,000	60	0.100	96	1196
101 - 500	300	95,000	317	0.400	?	?
501 - 1000	240	145,000	604	0.640	606	1572
1001 - 2000	185	260,000	1405	?	?	?
2001 - 4000	140	450,000	3214	0.965	1096	2171
4001 - 5000	15	66,000	4400	?	?	?
5001 - 10000	20	150,000	7500	1.000	1172	NA
Total	1000	1,172,000	1172			

Find a.) Fill in the highlighted values and graph the excess severities to identify the distribution.

b.) Assuming a Pareto distribution is appropriate, calculate the parameters of the distribution.

Solution

a.)

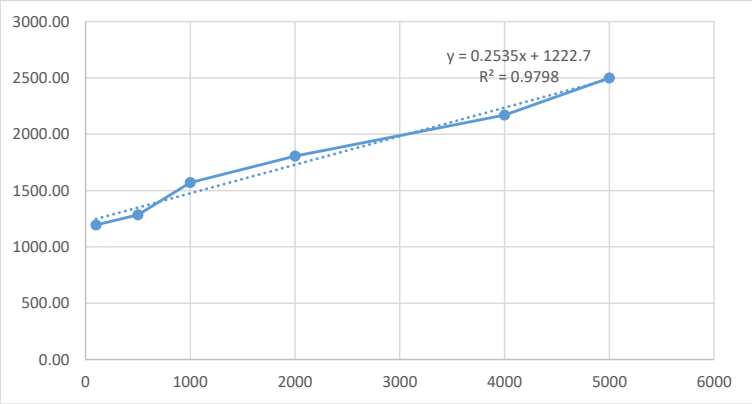
Group	# Claims	Total Loss	Severity	$F_{1000}(x)$	$E_{1000}[\hat{X};x]$	$e_{1000}(x)$
0 - 100	100	6000	60	0.1	96	1195.56
101 - 500	300	95000	317	0.4	401	1285.00
501 - 1000	240	145000	604	0.64	606	1572.22
1001 - 2000	185	260000	1405	0.825	856	1805.714
2001 - 4000	140	450000	3214	0.965	1096	2171.429
4001 - 5000	15	66000	4400	0.980	1122	2500
5001 - 10000	20	150000	7500	1	1172	NA
Total	1000	1172000	1172			

The details below are for the 1001 - 2000 claim size group

$F_{1000}(2000)$ = (100 + 300 + 240 + 185) / 1000
= 0.825

$E_{1000}[\hat{X};2000]$ = [(6000 + 95000 + 145000 + 260000) + (140 + 15 + 20)*2000] / 1000
= 856

$e_{1000}(2000)$ = (1172 - 856) / (1 - 0.825)
= 1805.71



b.) Since the graph of excess severities is very close to a straight line with positive slope, the underlying distribution is Pareto.

By equating the Pareto excess severity function with the equation for the linear regression trend line, we can estimate the parameters

$$e_{X(x)} = \frac{x+\beta}{\alpha-1} = 0.2535x + 1222.7$$

This gives $\frac{1}{\alpha-1} = 0.2535$ and $\frac{\beta}{\alpha-1} = 1222.7$

so $\alpha = 4.945$ and $\beta = 4823.27$