**Reading:** Bahnemann.Chapter6
**Model:** Source Text Example 6.3
**Problem Type:** Calculate ILFs loaded for expenses.

**Given**

Indemnity losses for a portfolio of insurance policies have a lognormal claim-size distribution with parameters

| | |
|---|---|
| μ = | 7 |
| σ = | 2.4 |

The policy per-claim limit applies only to the indemnity portion of a claim.

| | |
|---|---|
| 2,200 | Average per-claim loss adjustment expense |
| 0.0005 | Claim frequency per exposure |
| 35% | Variable expenses as a percentage of premium |
| 100,000 | Basic policy limit |

| L | E[X; L] |
|---|---|
| 100,000 | 8,896.04 |
| 1,000,000 | 15,345.22 |

**Find**

a.) Calculate the increased limit factor for a policy limit of $1,000,000

b.) For a policy with 400 exposures, calculate the premium at the
i.) Basic limit
ii.) $1,000,000 limit.

c.) Suppose instead loss adjustment expenses are 20.0% of the indemnity portion of the claim. Calculate:
i.) The ILF for a policy with $1,000,000 limit.
ii.) Basic policy premium
iii.) Policy premium for a policy with $1,000,000 limit.

**Useful Formulas**

Lognormal Distribution

$$E[X] = e^{\mu + \frac{\sigma^2}{2}}$$

$$E[X;L] = \mathrm{E[X]} \cdot \Phi\left(\frac{\log(L) - \mu - \sigma^2}{\sigma}\right) + \mathrm{L} \cdot \Phi\left(\frac{-\log(L) + \mu}{\sigma}\right)$$

**Solution**

The expected value of a lognormal distribution and its limited expected value are given by:

$$E[X] = e^{\mu+\frac{\sigma^2}{2}} \text{ and } E[X;L] = \mathrm{E[X]} \cdot \Phi\left(\frac{\log(L)-\mu-\sigma^2}{\sigma}\right) + \mathrm{L} \cdot \Phi\left(\frac{-\log(L)+\mu}{\sigma}\right)$$

a.) Since we're given the dollar amount of the loss adjustment expenses we'll use the second formulation for the ILF

$$I(L) = \frac{E[X;L]+\epsilon}{E[X;b]+\epsilon}$$

E[X;100,000] = 8,896.04
E[X;1,000,000] = 15,345.22
ε = 2,200
So I($1,000,000) = (15345.22 + 2200) / (8896.04 + 2200)
= 1.5812

b i.)
E[N] = m*ϕ = 400 * 0.0005 = 0.2
E[X;b] = 8896.04
Expected Loss Cost = mp = $E[N] \cdot (E[X;b]+\epsilon) =$ 2,219.21

Since we're not given any information about fixed expenses, we assume they are 0 and use a loss cost multiplier.
Basic Limit Premium = mp / (1 - v) = 2219.21 / (1 - 35.0%) = 3,414.17

b ii.)
E[N] = 0.2 (from part b.i.)
E[X;L] = 15,345.22
Expected Loss Cost = mp = $E[N] \cdot (E[X;L]+\epsilon) =$ 3509.04
$1,000,000 Limit Premium = mp / ( 1 - v )
= 5,398.53

But it's much quicker to apply the ILF I($1 million) from part a.
$1,000,000 Limit Premium = $P_b \cdot I(L)$ (remember $P_b$ is the basic limit premium)
= 3414.17 * 1.5812
= 5,398.49

(Minor differences due to rounding)

c i.)
I($1,000,000) = $\frac{E[X;L]\cdot(1+\mu)}{E[X;b]\cdot(1+\mu)}$ (notice the ALAE expense cancels out)
= 1.7249

c ii.)
E[N] = 0.2 (from part b.i.)
E[X;b] = 8,896.04
Expected Loss Cost = mp = $E[N] \cdot E[X;b] \cdot (1+\mu)$
= 2,135.05

Basic Premium = 3,284.69 (apply LCM as no fixed expenses)

c iii.) $1,000,000 Limit Premium = $P_b \cdot I(L)$
= 5,665.93

**Reading:** Bahnemann.Chapter6
**Model:** Source Text Example 6.3
**Problem Type:** Calculate ILFs loaded for expenses.

**Given**

Indemnity losses for a portfolio of insurance policies have a lognormal claim-size distribution with parameters

| | |
|---|---|
| μ = | 8 |
| σ = | 1.5 |

The policy per-claim limit applies only to the indemnity portion of a claim.

| | |
|---|---|
| 2,900 | Average per-claim loss adjustment expense |
| 0.0001 | Claim frequency per exposure |
| 37% | Variable expenses as a percentage of premium |
| 150,000 | Basic policy limit |

| L | E[X; L] |
|---|---|
| 150,000 | 8,635.26 |
| 500,000 | 9,086.69 |

**Find**

a.) Calculate the increased limit factor for a policy limit of $500,000

b.) For a policy with 504 exposures, calculate the premium at the
i.) Basic limit
ii.) $500,000 limit.

c.) Suppose instead loss adjustment expenses are 24.0% of the indemnity portion of the claim. Calculate:
i.) The ILF for a policy with $500,000 limit.
ii.) Basic policy premium
iii.) Policy premium for a policy with $500,000 limit.

**Useful Formulas**

Lognormal Distribution

$$E[X] = e^{\mu + \frac{\sigma^2}{2}}$$

$$E[X;L] = \mathrm{E[X]} \cdot \Phi\left(\frac{\log(L) - \mu - \sigma^2}{\sigma}\right) + \mathrm{L} \cdot \Phi\left(\frac{-\log(L) + \mu}{\sigma}\right)$$

**Solution**

The expected value of a lognormal distribution and its limited expected value are given by:

$$E[X] = e^{\mu+\frac{\sigma^2}{2}} \text{ and } E[X;L] = \mathrm{E[X]} \cdot \Phi\left(\frac{\log(L) - \mu - \sigma^2}{\sigma}\right) + \mathrm{L} \cdot \Phi\left(\frac{-\log(L) + \mu}{\sigma}\right)$$

a.) Since we're given the dollar amount of the loss adjustment expenses we'll use the second formulation for the ILF

$$I(L) = \frac{E[X;L] + \epsilon}{E[X;b] + \epsilon}$$

E[X;150,000] = 8,635.26
E[X;500,000] = 9,086.69
ε = 2,900
So I($500,000) = (9086.69 + 2900) / (8635.26 + 2900)
= 1.0391

b i.) E[N] = m*ϕ = 504 * 0.0001 = 0.0504
E[X;b] = 8635.26
Expected Loss Cost = mp = $E[N] \cdot (E[X;b] + \epsilon) =$ 581.38

Since we're not given any information about fixed expenses, we assume they are 0 and use a loss cost multiplier.
Basic Limit Premium = mp / (1 - v) = 581.38 / (1 - 37.0%) = 922.83

b ii.) E[N] = 0.0504 (from part b.i.)
E[X;L] = 9,086.69
Expected Loss Cost = mp = $E[N] \cdot (E[X;L] + \epsilon) =$ 604.13
$500,000 Limit Premium = mp / ( 1 - v )
= 958.94

But it's much quicker to apply the ILF I($1 million) from part a.
$500,000 Limit Premium = $P_b \cdot I(L)$ (remember $P_b$ is the basic limit premium)
= 922.83 * 1.0391
= 958.91

(Minor differences due to rounding)

c i.) I($500,000) = $\frac{E[X;L] \cdot (1+\mu)}{E[X;b] \cdot (1+\mu)}$ (notice the ALAE expense cancels out)
= 1.0523

c ii.) E[N] = 0.0504 (from part b.i.)
E[X;b] = 8,635.26
Expected Loss Cost = mp = $E[N] \cdot E[X;b] \cdot (1+\mu)$
= 539.67

Basic Premium = 856.62 (apply LCM as no fixed expenses)

c iii.) $500,000 Limit Premium = $P_b \cdot I(L)$
= 901.40