BATTLEACTS

Exam 8 PowerPack
Fundamentals ToolKit PDF

GLM

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Couret & Venter

Multi-dimensional Credibility	- Apply the multi-dimensional credibility technique
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NCCI Experience Rating

Experience Modification	- Calculate the NCCI experience modification

Mahler

Chi-squared testing	- Apply the chi-squared test for shifting risk parameters
Mean-Squared Error (MSE)	- Use the MSE to determine the optimal credibility
Accident Year Weights	- Determine the optimal least squares accident year weights

NCCI Circular

Basic Premium Factor	- Calculate the basic premium factor
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Discretize Severity Distribution	- Discretize the per-claim severity distribution

GLM_ExampleCalc (Problem 1)

Reading: GLM.Basics
Model: Source text
Problem Type: Predict auto claim severity using a GLM

Given

y	Target variable = loss cost
x_1	Driver age (predictor)
x_2	Marital status (predictor)
log	Link function
Gamma	Distribution

<= Model specification for GLM software, input along with a data set of observations.

<= We assume the loss cost after accounting for the predictors is random and follows a Gamma distribution.

Coefficient	Parameter
5.8	β_0 (Intercept)
0.1	β_1 (Coefficient for driver age)
-0.15	β_2 (Coefficient for marital status)
0.3	ϕ (Dispersion parameter)

<= GLM Software output

Find

a.) Predict the average claim severity for:
- i.) A 25-year old married driver
- ii.) A 35-year old unmarried driver

b.) Calculate the variance of the loss cost for:
- i.) A 25-year old married driver
- ii.) A 35-year old unmarried driver

Solution

To begin we need to understand the types of predictor variables used in the GLM. To do this, look at the model output.

Marital status is clearly a categorical variable as there isn't a continuous range of marital statuses. Looking at the model output, since there is only one coefficient (β_2) for marital status, we infer marital status is a binary variable, so either 1 or 0.

We're dependent on the question to specify which marital status corresponds to 0 and 1 respectively. Since it isn't explicitly called out, assume since most people are unmarried, that 0 = unmarried and 1=married. (This also matches with the logic of 1 = True and 0 = False.)

Next, driver age could be treated as either a continuous or discrete/categorical variable as we typically measure age in a whole number of years. Since the GLM output only has one coefficient for driver age (β_1) we infer age is a continuous variable as otherwise there would be a coefficient $\beta_{1,i}$ for each age in the data set.

Now we understand the GLM output, we can set up the GLM equation as follows: $g(\mu_i) = \ln(\mu_i) = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2$

Here we're using the natural logarithm for the log-link function g.

Now it's a matter of plugging in the numbers and then inverting the link function

a.) i.) $g(\mu_i)$ = 5.8 + 0.10 * 25 + -0.15 * 1 **<= Remember this driver is married so marital status = 1**
= 8.15
Inverting the link function by exponentiating gives
μ_i = 3,463.38 **<= This is the predicted average loss cost for a claim for the set of married 25-year old drivers**

a.) ii.) $g(\mu_i)$ = 5.8 + 0.10 * 35 + -0.15 * 0
= 9.3
Inverting the link function by exponentiating gives
μ_i = 10,938.02 **<= This is the predicted average loss cost for a claim for the set of unmarried 35-year old drivers**

Notice how we could also write this as $\mu_i = e^{\beta_0} \cdot e^{\beta_1 \cdot x_1} \cdot e^{\beta_2 \cdot x_2}$
In a.)i.) above this becomes μ_i = 330.30 * 12.182 * 0.861

We can split this apart as:

330.30	is the "base rate" – the average severity for the whole book of business/data set
12.182	is the factor for a driver aged 25
0.861	is the factor for a married driver

We can further interpret the results of a.) as follows:
a.) i.) The severity distribution for the set of married 25-year old drivers follows a Gamma distribution with μ = 3,463.38 and ϕ = 0.3
a.) ii.) The severity distribution for the set of unmarried 35-year old drivers follows a Gamma distribution with μ = 10,938.02 and ϕ = 0.3

Notice in both cases we have ϕ = 0.3. This is because ϕ is assumed to be constant across the entire data set.

b.) We now have fully specified Gamma distributions for part a.) so we can calculate the variance as ϕ * V(μ), which for a Gamma distribution is $\phi * \mu^2$

b. i.) Variance = 0.3 * 3,463.38 ^2 = 3,598,498.37
b. ii.) Variance = 0.3 * 10,938.02 ^2 = 35,892,079.26

The higher-risk driver (determined by the average claim severity, μ_i) has a higher variance than the lower risk driver despite ϕ being constant.

Reading: GLM.Basics
Model: 2013.Q2
Problem Type: Define the design matrix and vector of responses

Given An actuary is building a log-link generalized linear model to create a Homeowners Hurricane Severity model using the data below.

Dollars of loss

Hurricane Shutters	Distance to Coast	
	≤ 25 miles	> 25 miles
Yes	\$5,968,000	\$5,092,000
No	\$5,609,000	\$1,133,000

Number of Claims

Hurricane Shutters	Distance to Coast	
	≤ 25 miles	> 25 miles
Yes	14	35
No	23	18

Average Coverage A Amount

Hurricane Shutters	Distance to Coast	
	≤ 25 miles	> 25 miles
Yes	\$446,000	\$350,000
No	\$251,000	\$269,000

The model will include four parameters: β_0, β_1, β_2, and β_3, where β_0 is the intercept, β_1 is the average severity for homes with hurricane shutters, β_2 is the average severity for homes greater than 25 miles from the coast, and β_3 is the average severity for the natural log of the average Coverage A Amount (continuous variable).

Find

a. Define the design matrix [X].

b. Define the vector of responses [Y].

Solution

Average Severity = Dollars of loss / Number of Claims

Average Severity

Hurricane	Distance to Coast	
Shutters	≤ 25 miles	> 25 miles
Yes	$426,286	$145,486
No	$243,870	$62,944

We have four distinct data points in the GLM, one for each combination of Hurricane Shutters and Distance to Coast. As such, the design matrix will have four rows.

Alice: "Although there are only four records in the data set, remember these likely came from many observations that were aggregated to this level."

The design matrix consists of a column for each parameter. In the case of a categorical variable the value is either 1 or 0 depending on whether or not the record has that rating characteristic. For a continuous variable, the actual value is used after any transformation needed.

$$X = \begin{array}{cccc} \beta0 & \beta1 & \beta2 & \beta3 \\ \hline 1 & 1 & 0 & \mathrm{LN}(446{,}000) \\ 1 & 1 & 1 & \mathrm{LN}(350{,}000) \\ 1 & 0 & 0 & \mathrm{LN}(251{,}000) \\ 1 & 0 & 1 & \mathrm{LN}(269{,}000) \end{array}$$

Alice: "Notice there's a row for every observation and all rows have a 1 in the intercept column (β_0)."

Alice: "It's important you follow the given description of the parameters because this tells you the base levels. Here it's implicit the base levels are:

1. Homes with no hurricane shutters

2. Homes less than or equal to 25 miles from the coast.

Remember the base level is usually the one with the most exposures. This makes sense here as people tend to live close to the coast and not always have hurricane shutters."

The vector of responses is a column vector of the average severities. It's important you write them down in the same order you processed the records when building the design matrix.

$$Y = \begin{array}{c} \$426{,}286 \\ \$145{,}486 \\ \$243{,}870 \\ \$62{,}944 \end{array}$$

GLM_Offsets (Problem 1)

Reading: GLM.Basics
Model: Source text
Problem Type: Offset deductibles in a GLM

Given A loss elimination ratio (LER) analysis was performed to calculate relativities for Auto Collision deductibles.

Deductible	Factor
\$500	1.000
\$1,000	0.900
\$1,500	0.830

← This is the base level as it has a relativity of 1.000

A GLM is being built to model collision pure premium. The GLM will use a Gamma distribution with a log-link function.

Find a.) Briefly explain how the modeler can account for the impact of the insured's choice of deductible.

b.) The modeler was also provided with a curve which relates household income to collision pure premiums.
Briefly describe how the modeler can also account for this data in the model.

Solution

a.) The modeler can offset the deductible. This is done as follows:

1.) Transform the deductible relativities to the same scale as the link function.
Here, the log-link function is used, so we get

Deductible	Factor	Log(Factor)
\$500	1.000	0.000
\$1,000	0.900	-0.105
\$1,500	0.830	-0.186

2.) Add this to the **linear predictor**, i.e. the right hand side of the GLM equation: $g(\mu_i) = \ln(\mu_i) = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2$

That is, $g(\mu_i) = \ln(\mu_i) = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + 1 \cdot \text{offset}_i$

Do this for each record in the data set.
Offset$_i$ is 0 if the record had a \$500 deductible, -0.105 if it had a \$1,000 deductible, and -0.186 if it had a \$1,500 deductible.

b.) The modeler can offset the household income curve as well as the deductible in the model.
It is important to match the scale of each offset to the link function. In this case, we would take the log of the household income for each record in the data set. When there are two or more variables to be offset in the model, the offsets may be added together.

For example, suppose a record in the data set has a \$1,000 collision deductible and a household income of \$75,000.
Further, when the household income is applied to the curve, it results in a factor of 1.025.
The offset for this record would be ln(0.9) + ln(1.025) = -0.081

GLM_Quantiles (Problem 1)

Reading: GLM.Validation
Model: Source Text
Problem Type: Perform a quantiles test

Given

Observation	Predicted Pure Premium	Actual Premium Premium
1	$362	$628
2	$780	$635
3	$849	$306
4	$138	$126
5	$343	$239
6	$989	$835
7	$381	$515
8	$716	$143
9	$696	$738
10	$685	$388

Find Create a quantiles plot using quintiles.

Solution First rank the observations according to their predicted pure premium

Observation	Predicted Pure Premium	Actual Premium Premium	Rank
1	$362	$628	3
2	$780	$635	8
3	$849	$306	9
4	$138	$126	1
5	$343	$239	2
6	$989	$835	10
7	$381	$515	4
8	$716	$143	7
9	$696	$738	6
10	$685	$388	5

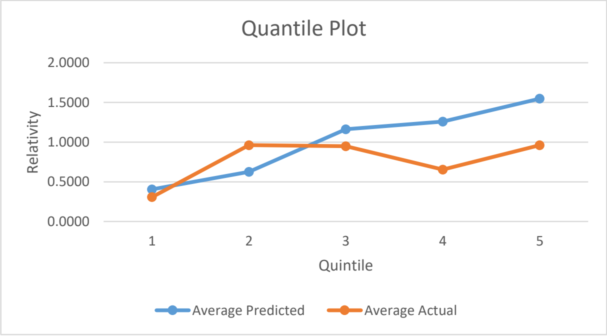
We'll assume each observation is one exposure since we're not given any information about weights.
Since we're asked for quintiles, we'll need five groups. Since each observation is one exposure, we'll have two observations per quintile.

Quintile	Average Predicted	Average Actual	Observations
1	$241	$183	4, 5
2	$372	$572	1, 7
3	$691	$563	10, 9
4	$748	$389	8, 2
5	$919	$571	3, 6
Overall	$594	$455	NA

The normalized values are found by dividing by the average overall predicted pure premium

Quintile	Average Predicted	Average Actual
1	0.4050	0.3073
2	0.6255	0.9623
3	1.1627	0.9480
4	1.2595	0.6550
5	1.5474	0.9606
Overall	1.0000	0.7666

The predicted values are plotted on the x-axis and the actual values on the y-axis.



Reading: GLM.Validation

Model: Source text

Problem Type: Produce a double lift chart using the data provided

Given An actuary wants to compare two Homeowners loss cost models. They have scored each model using the following records.

Record #	Model A Loss Cost	Model B Loss Cost	Actual Loss Cost
1	$1,080	$1,000	$990
2	$1,210	$1,280	$1,170
3	$1,590	$1,460	$1,490
4	$1,130	$970	$1,230
5	$1,320	$1,240	$1,330
6	$920	$830	$920
7	$930	$780	$920
8	$1,360	$1,460	$1,350
9	$860	$740	$870
10	$730	$570	$660

Find Using quintiles, produce the standard double lift chart and alternate double lift chart then recommend a model.

Solution We're asked to use quintiles so we need to first calculate the sort ratio for each record and then group into five groups by ranking the sort ratio in ascending order.

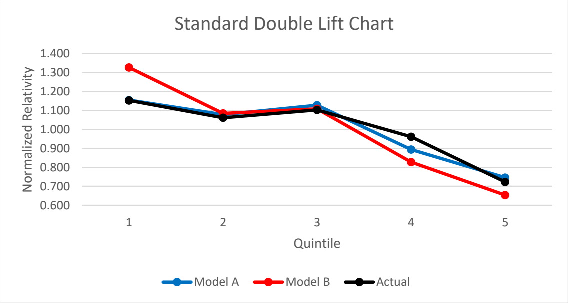
Record #	Model A Loss Cost	Model B Loss Cost	Actual Loss Cost	Sort Ratio	Rank
1	$1,080	$1,000	$990	1.080	4
2	$1,210	$1,280	$1,170	0.945	2
3	$1,590	$1,460	$1,490	1.089	5
4	$1,130	$970	$1,230	1.165	8
5	$1,320	$1,240	$1,330	1.065	3
6	$920	$830	$920	1.108	6
7	$930	$780	$920	1.192	9
8	$1,360	$1,460	$1,350	0.932	1
9	$860	$740	$870	1.162	7
10	$730	$570	$660	1.281	10

Quintile	Contains Ranks	Model A Avg Loss Cost	Model B Avg Loss Cost	Actual Avg Loss Cost
1	1, 2	$1,285	$1,370	$1,260
2	3, 4	$1,200	$1,120	$1,160
3	5, 6	$1,255	$1,145	$1,205
4	7, 8	$995	$855	$1,050
5	9, 10	$830	$675	$790
Total		**$1,113**	**$1,033**	**$1,093**

Next, normalize each column using the Total row

Quintile	Model A	Model B	Actual
1	1.155	1.326	1.153
2	1.078	1.084	1.061
3	1.128	1.108	1.102
4	0.894	0.828	0.961
5	0.746	0.653	0.723

We can now plot the normalized figures to produce the standard double lift chart.



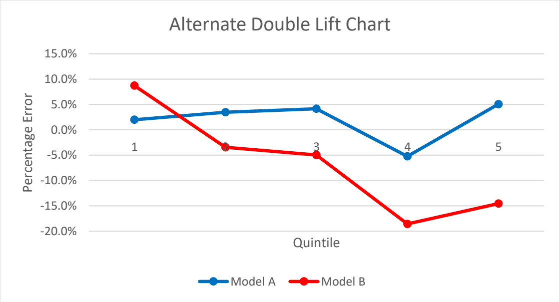
In the standard double lift chart, Model A tracks the actual results more closely than Model B, i.e. Model A is the better model.
In the alternate view, Model A has a flatter line than Model B, so Model A better approximates the actual pure premiums.

Recommend using Model A.

To produce the alternate view, we calculate the percentage error for each quintile.

	% Error	
Quintile	Model A	Model B
1	2.0%	8.7%
2	3.4%	-3.4%
3	4.1%	-5.0%
4	-5.2%	-18.6%
5	5.1%	-14.6%

$$\%\ \text{Error} = \frac{\text{Model Pure Premium}}{\text{Actual Pure Premium}} - 1$$



Reading: GLM.Validation
Model: Source Text
Problem Type: Investigate which rating plan performs best using a loss ratio chart.

Given A GLM is used to produce a new rating plan and its performance is measured using a holdout sample of 30 risks. Each risk represents a single exposure.

Risk	Current Premium	Actual Loss	Predicted Loss
1	1,374	709	794
2	1,754	1,443	1,558
3	158	169	147
4	1,080	520	577
5	3,371	1,599	1,775
6	1,366	1,326	1,313
7	1,178	1,008	907
8	1,575	748	695
9	2,974	1,391	1,391
10	160	163	141
11	1,083	572	492
12	3,691	1,950	1,794
13	1,005	975	1,004
14	1,183	1,131	961
15	691	598	520
16	2,175	1,937	1,782
17	1,782	1,781	1,781
18	1,738	1,430	1,530
19	1,435	1,352	1,284
20	2,298	1,892	2,175
21	2,880	1,463	1,638
22	1,594	774	696
23	1,677	1,651	1,866
24	877	826	925
25	118	117	103
26	2,915	1,554	1,460
27	211	182	189
28	1,458	1,554	1,647
29	392	345	351
30	1,663	813	837

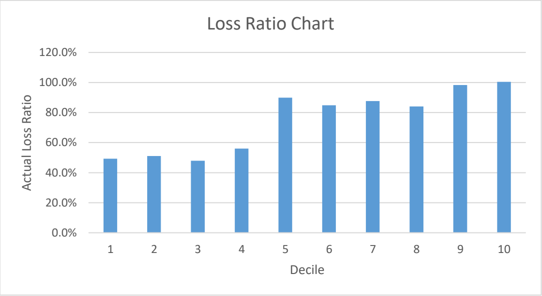
Find Use a loss ratio chart with deciles to demonstrate whether the new plan represents an improvement over the current plan.

Solution

First compute the predicted loss ratio as Predicted Loss / Current Premium.
Then order the resulting table by increasing predicted loss ratio

Risk	Current Premium	Actual Loss	Predicted Loss Ratio	Quantile
22	1,594	774	43.7%	1
8	1,575	748	44.1%	1
11	1,083	572	45.4%	1
9	2,974	1,391	46.8%	2
12	3,691	1,950	48.6%	2
26	2,915	1,554	50.1%	2
30	1,663	813	50.3%	3
5	3,371	1,599	52.7%	3
4	1,080	520	53.4%	3
21	2,880	1,463	56.9%	4
1	1,374	709	57.8%	4
15	691	598	75.3%	4
7	1,178	1,008	77.0%	5
14	1,183	1,131	81.2%	5
16	2,175	1,937	81.9%	5
25	118	117	87.3%	6
18	1,738	1,430	88.0%	6
10	160	163	88.1%	6
2	1,754	1,443	88.8%	7
19	1,435	1,352	89.5%	7
29	392	345	89.5%	7
27	211	182	89.6%	8
3	158	169	93.0%	8
20	2,298	1,892	94.6%	8
6	1,366	1,326	96.1%	9
13	1,005	975	99.9%	9
17	1,782	1,781	99.9%	9
24	877	826	105.5%	10
23	1,677	1,651	111.3%	10
28	1,458	1,554	113.0%	10

Quantile	Actual Loss	Current Premium	Actual Loss Ratio
1	2,094	4,252	49.2%
2	4,895	9,580	51.1%
3	2,932	6,114	48.0%
4	2,770	4,945	56.0%
5	4,076	4,536	89.9%
6	1,710	2,016	84.8%
7	3,140	3,581	87.7%
8	2,243	2,667	84.1%
9	4,082	4,153	98.3%
10	4,031	4,012	100.5%



As we view the deciles from left to right the loss ratios are generally increasing which means the proposed model performs better than the current model.

Reading: GLM.Validation
Model: Source Text
Problem Type: Confusion matrix

Given

An insurance company wants to make sure its litigation claims get assigned to a senior claims rep as soon as possible.
A logistic model was built to predict the likelihood of a claim going to litigation.

Claim Number	Claim went to Litigation	Predicted Probability of going to Lit
1	Y	96%
2	N	13%
3	Y	37%
4	N	52%
5	N	96%
6	N	21%
7	Y	50%
8	N	28%
9	N	79%
10	Y	91%
11	N	17%
12	Y	91%

Find

Calculate confusion matrices for discrimination thresholds of 0.3 and 0.55.

Solution

Claim Number	Claim went to Litigation	Predicted Probability of going to Lit	Discriminant Threshold	
			0.30	0.55
1	Y	0.96	TP	TP
2	N	0.13	TN	TN
3	Y	0.37	TP	FN
4	N	0.52	FP	TN
5	N	0.96	FP	FP
6	N	0.21	TN	TN
7	Y	0.50	TP	FN
8	N	0.28	TN	TN
9	N	0.79	FP	FP
10	Y	0.91	TP	TP
11	N	0.17	TN	TN
12	Y	0.91	TP	TP

Here, TP means True Positive, TN means True Negative, FP means False Positive, and FN means False Negative.

We assign these values as follows:
TP -> Claim went to Litigation = Y and Predicted Probability > Discriminant Threshold
FP -> Claim went to Litigation = N and Predicted Probability > Discriminant Threshold
FN -> Claim went to Litigation = Y and Predicted Probability < Discriminant Threshold
TN -> Claim went to Litigation = N and Predicted Probability < Discriminant Threshold

General Confusion Matrix

		Predicted	
		Positive	Negative
Actual	Positive	TP	FN
	Negative	FP	TN

We assign the count of each type to the matrix.

Threshold = 0.3

5	0
3	4

Threshold = 0.55

3	2
2	5

Observe the lower threshold has less false negatives and more false positives than the higher threshold.
Whether this is a good or bad thing depends on the scenario.
If the price of a false positive is low in terms of money/time/resources and the cost of missing a true positive is high then this is good.
If it is the reverse, i.e. little gain for a lot of cost then this is bad.

GLM_Sensitivity (Problem 1)

Reading: GLM.Validation
Model: Source Text
Problem Type: Calculate the sensitivity, specificity and false positive rate.

Given The following confusion matrix:

	Predicted	
Actual	6	1
	2	4

Find Calculate the sensitivity, specificity, and false positive rate.

Solution First relate the given confusion matrix to the general layout of a confusion matrix

$$\begin{array}{c|cc} & \multicolumn{2}{c}{\text{Predicted}} \\ \hline \text{Actual} & 6 & 1 \\ & 2 & 4 \end{array} = \begin{array}{cc} \text{TP} & \text{FN} \\ \text{FP} & \text{TN} \end{array}$$

Sensitivity = TP / (TP + FN) = 0.85714

Specificity = TN / (TN + FP) = 0.67

False positive rate = 1 - Specificity = 0.33

Alice: "To help recall the denominator, notice it has each of the four letters exactly once and it begins with the term in the numerator. Further, on the exam, make sure you clearly label the confusion matrix to show which are actual values and which are predicted. Depending on which text/online source you read these may be switched. Here we're presenting the material in the same way as the GLM text."

Reading: GLM.Validation

GLM_LogisticLorenz (Problem 1)

Model: Source text

Problem Type: Produce a table which can be used to plot the Lorenz curve for the output of a logistic regression.

Given An actuary has built a logistic regression model on a training data set and has run the model on a holdout sample of 50 observations. The model produced the following data:

Table of Predictions

0.219	0.740
0.342	0.080
0.860	0.798
0.429	0.086
0.803	0.023
0.634	0.960
0.432	0.325
0.284	0.994
0.888	0.384
0.110	0.943
0.772	0.421
0.695	0.391
0.318	0.424
0.375	0.405
0.521	0.830
0.963	0.652
0.329	0.489
0.524	0.264
0.496	0.384
0.072	0.790
0.328	0.523
0.199	0.448
0.640	0.673
0.553	0.554
0.139	0.164

Associated Outcomes*

FALSE	TRUE
FALSE	FALSE
TRUE	TRUE
TRUE	FALSE
TRUE	FALSE
TRUE	TRUE
FALSE	FALSE
FALSE	TRUE
TRUE	FALSE
FALSE	TRUE
TRUE	FALSE
TRUE	TRUE
FALSE	FALSE
TRUE	FALSE
TRUE	TRUE
TRUE	TRUE
FALSE	TRUE
FALSE	TRUE
FALSE	FALSE
FALSE	TRUE
FALSE	FALSE
FALSE	FALSE
FALSE	FALSE
TRUE	TRUE
FALSE	FALSE

* TRUE means the event occurred while FALSE means it didn't.

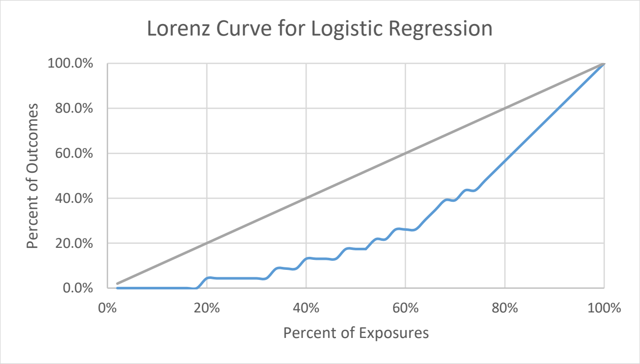
Find Produce a table which can be used to plot the Lorenz curve for the output of a logistic regression.

Solution

We must order the risks by increasing prediction and then plot the cumulative exposure on the x-axis and the cumulative percentage of true outcomes on the y-axis.

Ranked Outcomes	
9	39
16	3
45	42
24	4
43	1
34	48
25	13
11	50
46	18
5	47
40	22
38	20
12	23
17	21
29	44
49	36
15	27
31	10
28	19
2	41
14	30
8	26
35	37
32	33
6	7

			Cumulative	
Rank	Prediction	Outcome	Exposure	TRUE
1	0.023	FALSE	2%	0.0%
2	0.072	FALSE	4%	0.0%
3	0.080	FALSE	6%	0.0%
4	0.086	FALSE	8%	0.0%
5	0.110	FALSE	10%	0.0%
6	0.139	FALSE	12%	0.0%
7	0.164	FALSE	14%	0.0%
8	0.199	FALSE	16%	0.0%
9	0.219	FALSE	18%	0.0%
10	0.264	TRUE	20%	4.3%
11	0.284	FALSE	22%	4.3%
12	0.318	FALSE	24%	4.3%
13	0.325	FALSE	26%	4.3%
14	0.328	FALSE	28%	4.3%
15	0.329	FALSE	30%	4.3%
16	0.342	FALSE	32%	4.3%
17	0.375	TRUE	34%	8.7%
18	0.384	FALSE	36%	8.7%
19	0.384	FALSE	38%	8.7%
20	0.391	TRUE	40%	13.0%
21	0.405	FALSE	42%	13.0%
22	0.421	FALSE	44%	13.0%
23	0.424	FALSE	46%	13.0%
24	0.429	TRUE	48%	17.4%
25	0.432	FALSE	50%	17.4%
26	0.448	FALSE	52%	17.4%
27	0.489	TRUE	54%	21.7%
28	0.496	FALSE	56%	21.7%
29	0.521	TRUE	58%	26.1%
30	0.523	FALSE	60%	26.1%
31	0.524	FALSE	62%	26.1%
32	0.553	TRUE	64%	30.4%
33	0.554	TRUE	66%	34.8%
34	0.634	TRUE	68%	39.1%
35	0.640	FALSE	70%	39.1%
36	0.652	TRUE	72%	43.5%
37	0.673	FALSE	74%	43.5%
38	0.695	TRUE	76%	47.8%
39	0.740	TRUE	78%	52.2%
40	0.772	TRUE	80%	56.5%
41	0.790	TRUE	82%	60.9%
42	0.798	TRUE	84%	65.2%
43	0.803	TRUE	86%	69.6%
44	0.830	TRUE	88%	73.9%
45	0.860	TRUE	90%	78.3%
46	0.888	TRUE	92%	82.6%
47	0.943	TRUE	94%	87.0%
48	0.960	TRUE	96%	91.3%
49	0.963	TRUE	98%	95.7%
50	0.994	TRUE	100%	100.0%



Alice: "As far as we know it's still unlikely you'll be asked to graph something in the exam environment. It's probably reasonable to create a table of data for a graph though."

"Also, this type of problem is a lot easier if you have the XLOOKUP function in Excel. However, not all versions of Excel have this functionality so we've shown a way of doing this another way. Pearson claims to have XLOOKUP in their software so you could practice with a small example in their online practice environment."

Reading: Holmes.Intro
Model: Source Text
Problem Type: Offsets, linear predictors, and rating tables

Given

An actuary is building a basic homeowners pure premium GLM using the following variables:
Age of Home which takes integer values between 0 and 10 (inclusive) but is modeled as a continuous variable.
NoFireExtinguisherInd which is 1 if **no** fire extinguisher is present in the home and 0 otherwise.
The base level for *NoFireExtinguisherInd* is 0 (more homes have a fire extinguisher than do not).

The actuary is using a Tweedie distribution with the log link function.

The results of their GLM are shown below.

	Estimate
(Intercept)	4.605
Age of Home	0.010
NoFireExtinguisherInd:1	0.182

You may assume all variables are statistically significant and should be included in the model.

The actuary has performed a separate loss elimination ratio analysis to price their deductible offerings:

Deductible	Discount / Surcharge
$500	15%
$1,000	0%
$2,000	-10%

Find

a.) What is the value of the linear predictor for a home that is 2 years old without a fire extinguisher and having a $500 deductible?

b.) Construct rating tables for the *Age of Home* and *NoFireExtinguisherInd* rating variables.

c.) Briefly describe two potential weaknesses of the actuary's GLM in relation to the *Age of Home* rating variable.

Solution

a.) First we should translate the deductible discount/surcharge into a table of relativities and then adjust them to match the scale of the GLM.

Deductible	Relativity	Offset ln(Rel)
$500	1.150	0.1398
$1,000	1.000	0.0000
$2,000	0.900	-0.1054

Linear predictor $g(\mu_i) = \beta_0 + \beta_1 \cdot Age\ of\ Home_i + \beta_2 \cdot NoFireExtinguisherInd_i + Offset_i$

β_0 = 4.605
β_1 = 0.010
β_2 = 0.182

$g(\mu_i)$ = 4.9468 = 4.605 + 0.010 * 2 + 0.182 * 1 + 0.1398

b.) The inverse of the log link function, ln(μ), is exp(μ).

$$\mu_i = \exp(\beta_0 + \beta_1 \cdot Age\ of\ Home_i + \beta_2 \cdot NoFireExtinguisherInd_i + Offset_i)$$
$$= e^{\beta_0} \cdot \left(e^{\beta_1}\right)^{Age\ of\ Home_i} \cdot \left(e^{\beta_2}\right)^{NoFireExtinguisherInd_i} \cdot e^{Offset_i}$$

The multiplicative relativities for *Age of Home* and *Fire Extinguishers* are then

Age of Home	Relativity
0	1.000
1	1.010
2	1.020
3	1.030
4	1.041
5	1.051
6	1.062
7	1.073
8	1.083
9	1.094
10	1.105

Fire Extinguisher	Relativity
Yes	1.000
No	1.200

c.) Two potential weaknesses are:

- Age of Home was not logged so it doesn't match the scale of the link function. This is why we end up with Age of Home as a power rather than a direct multiplication.
- Age of Home only takes discrete integer values yet is modeled as a continuous variable via a first-order polynomial. It may be better to treat it as a categorical variable and possibly group some of the ages.

Reading: Fisher.ExperienceRating
Model: 2018.Q9
Problem Type: Apply the quintiles test

Given

Risk #	Manual Premium	Loss	Mod	Standard Premium
1	810	750	0.97	786
2	900	490	0.68	612
3	950	1,075	1.13	1,074
4	975	650	0.78	761
5	1,075	850	0.88	946
6	1,100	1,000	0.96	1,056
7	1,225	1,300	1.06	1,299
8	1,300	800	0.72	936
9	1,450	1,175	0.90	1,305
10	1,500	975	0.76	1,140

Find Apply the quintiles test

Solution

First rank the risks from smallest to largest experience modification

Rank	Risk #	Manual Premium	Loss	Mod	Standard Premium
1	2	900	490	0.68	612
2	8	1300	800	0.72	936
3	10	1500	975	0.76	1140
4	4	975	650	0.78	761
5	5	1075	850	0.88	946
6	9	1450	1175	0.90	1305
7	6	1100	1000	0.96	1056
8	1	810	750	0.97	786
9	7	1225	1300	1.06	1299
10	3	950	1075	1.13	1074

Next, collapse into five groups. Here it is natural to group into consecutive pairs - on the exam make sure to state your logic when grouping.

Risk #s	Quintile	Manual Premium (1)	Loss (2)	Manual LR (3)	Average Mod (4)	Standard Premium (5)	Standard LR (6)
2, 8	1	2200	1290	58.6%	0.70	1548	83.3%
10, 4	2	2475	1625	65.7%	0.77	1901	85.5%
5, 9	3	2525	2025	80.2%	0.89	2251	90.0%
6, 1	4	1910	1750	91.6%	0.96	1842	95.0%
7, 3	5	2175	2375	109.2%	1.09	2373	100.1%

(1), (2), (5) Sum over risks in quintile

(3) = (2) / (1)

(4) = Sumproduct of the experience mod and manual premium within quintile, divided by the sum of the manual premium in the quintile.

(6) = (2) / [(4) * (1)]

Notes:

1.) Since Standard Premium = Experience * Manual Premium for any given risk, it wasn't necessary to calculate the average experience modification factor for each quintile.

2.) We get the same result if we calculate (6) = (2) / (5).

Reading: Fisher.ExperienceRating

Model: 2011.Q16

Problem Type: Apply the Quintiles Test and interpret the results

Fisher_QuintilesTest2 (Problem 1)

Given

Quintile	Actual Losses	Expected Losses	Modified Expected Loss
1	187,000	190,000	182,000
2	195,000	195,000	187,000
3	201,000	200,000	195,000
4	227,000	205,000	210,000
5	238,000	210,000	255,000

Find Apply the Quintiles Test and interpret the results.

Solution

We aren't give the premium in each quintile, so we'll need to use the adjusted versions of the manual and standard loss ratios.
Also, we're already given the data in quintiles, so there is no need for the experience modification factor, we can presume the quintiles were calculated with them already sorted from smallest to largest.

$$\boxed{\text{Manual Loss Ratio} = \frac{\text{Actual Losses}}{\text{Expected Losses}}}$$

$$\boxed{\text{Standard Loss Ratio} = \frac{\text{Actual Losses}}{\text{Modified Expected Losses}}}$$

Quintile	Manual LR	Standard LR
1	98.4%	102.7%
2	100.0%	104.3%
3	100.5%	103.1%
4	110.7%	108.1%
5	113.3%	93.3%

Interpreting the results

Manual Loss Ratio Dispersion	14.9%	= 113.3% - 98.4%
Standard Loss Ratio Dispersion	14.8%	= 108.1% - 93.3%

There is an upward trend in the manual loss ratios so the plan does a good job at **identifying** differences between risks.

There is no noticeable trend in the standard loss ratios. However the values are not approximately equal for all risks and the dispersion is not materially lower than seen in the manual loss ratios. This implies the plan does **not do a good job of adjusting** for differences between risks.

Reading: Fisher.ExperienceRating
Model: 2018.Q9
Problem Type: Apply the efficiency test

Given

Insurer 1's Plan

Quintile	Manual Loss Ratio	Standard Loss Ratio
1	58.6%	83.3%
2	65.6%	85.5%
3	80.2%	90.0%
4	91.6%	95.0%
5	109.2%	100.1%

Insurer 2's Plan

Quintile	Manual Loss Ratio	Standard Loss Ratio
1	58.6%	94.5%
2	65.7%	90.0%
3	80.2%	85.3%
4	91.6%	79.7%
5	109.2%	75.3%
Sample Variance	0.0411	0.0059

Find Use the Efficiency Test to determine which experience rating plan is better.

Solution

$$\text{Efficiency Test Statistic} = \frac{\text{Standard Loss Ratio Sample Variance}}{\text{Manual Loss Ratio Sample Variance}}$$

Insurer 1's Plan

Quintile	Manual Loss Ratio	Standard Loss Ratio
1	58.6%	83.3%
2	65.6%	85.5%
3	80.2%	90.0%
4	91.6%	95.0%
5	109.2%	100.1%
Sample Variance	0.04118	0.00473

Alice: "The efficiency test is defined using sample variance even though you'll get the same answer if you use the population variance calculation instead. It's hard to say if the CAS would penalize you for using population variance in the exam."

Efficiency Test Statistic: = 0.00473 / 0.04118
= 0.1148

Insurer 2's Plan

Efficiency Test Statistic: = 0.00590 / 0.04110
= 0.1436

Since 0.1148 < 0.1436 Insurer 1's plan is better

Note: Here we are using the following formula for the sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Alice: "Notice here we're using the Var.S() Excel function. This is okay because we're not provided with any other information. However, if we were told the quintiles had different manual premium volumes then you need to calculate the sample variance by hand by finding the first and second moments. This is because Var.S() assumes all values have the same weight."

Reading: Fisher.RiskSharing

Model: Source Text

Problem Type: Calculate the expenses as a percentage of the guaranteed cost premium

Fisher_RS5 (Problem 1)

Given

Loss Conversion Factor	1.1
Expected Loss Ratio	0.7
Expense Ratio	0.2

Find Calculate the expense portion of the basic premium as a percentage of the guaranteed-cost premium.

Solution

The formula for the expense portion of the basic premium as a percentage of the guaranteed cost premium is: $e-(c-1)E$

We're given

c = Loss Conversion Factor	1.1
E = Expected Loss Ratio	0.7
e = Expense Ratio	0.2

Plugging these into the formula yields:

13.0% = 0.2 - (1.1 - 1)*0.7

Reading: Fisher.RiskSharing
Model: Source Text
Problem Type: Calculate the retrospective rating premium

Given

B =>	Basic Premium Amount	$150,000
c =>	Loss Conversion Factor	1.100
T =>	Tax Multiplier	1.031
	Per-Occurrence Limit	$100,000
	Maximum ratable loss	$500,000

Large Claims
$15,000
$25,000
$50,000
$100,000
$1,000,000

There are 15 claims on the policy. 10 of those claims are below the per-occurrence limit and total $25,000.
The other 5 claims have the following values:

Find Calculate the retrospective rating premium.

Solution

The retrospective rating formula is: $R = (B + cL) \cdot T$

We're given

B = Basic Premium Amount	$150,000
c = Loss Conversion Factor	1.100
T = Tax Multiplier	1.031

We need to calculate L, the ratable loss and then we may apply the formula.

To find L we must read the claims information carefully and apply the per-occurrence limit and then the maximum ratable loss constraint.

Evaluate each claim in turn and keep track of the cumulative claims so you can apply the maximum ratable loss condition.

Claim	Amount Below per-occurrence limit	Comments
First 10 claims	$25,000	We're told these are all individually below the per-occurrence limit.
$15,000	$15,000	
$25,000	$25,000	
$50,000	$50,000	
$100,000	$100,000	Capped by per-occurrence limit
$1,000,000	$100,000	Capped by per-occurrence limit
TOTAL	$315,000	

Now cap the total at the maximum ratable loss if it exceeds it.

Ratable Loss = $315,000 **<= L**

Finally, apply the retrospective rating formula

R = (150000 + 1.1 * 315000) * 1.031

= $511,892

Reading: Fisher.RiskSharing
Model: Source Text
Problem Type: Visualize the cash flow for both the policyholder and the insurer under an incurred retrospective rating plan.

Given

Pricing Assumptions	
$1,100,000	Initial Premium
$600,000	Expected Primary Loss & ALAE
$300,000	Expected Excess Loss & ALAE
$55,000	Commission
$15,000	General Expenses
$5,000	Underwriting Profit Provision
10.0%	ULAE
3.0%	Tax Rate

There is no aggregate excess loss exposure.

Payment Patterns

Time (Years)	Initial Premium	(1) Primary Incurred Loss & ALAE	(2) Primary Paid Loss & ALAE	(3) Excess Paid Loss & ALAE	(4) Total Paid Loss & ALAE	Commission	(5) General Expenses	(6) ULAE
0.00	100%					100%	25.0%	
0.25		10.7%	2.1%	0.1%	1.4%		43.8%	7.3%
0.50		26.3%	7.2%	0.5%	5.0%		62.5%	16.2%
0.75		45.4%	14.5%	2.0%	10.3%		81.3%	26.5%
1.00		65.5%	23.4%	5.0%	17.3%		100.0%	38.0%
1.50		77.3%	40.9%	15.0%	32.3%			49.2%
2.50		87.9%	63.5%	35.0%	54.0%			65.5%
3.50		93.9%	79.8%	60.0%	73.2%			79.9%
4.50		97.4%	90.4%	80.0%	86.9%			90.2%
5.50		98.9%	95.6%	90.0%	93.7%			95.3%
6.50		99.7%	97.7%	95.0%	96.8%			97.6%
7.50		100.0%	100.0%	100.0%	100.0%			100.0%

Find The incurred retrospective rating plan basic premium at each point in time and illustrate the cash flows from both the policyholder and insurer perspectives.

Note As the CAS moves towards computer based testing, this type of problem (which is tedious to do by hand) becomes much easier to test.

Solution

The loss conversion factor is c = 1 + ULAE %, i.e.

c = 1.100

Now let's calculate the basic premium which should cover the converted expected excess loss and ALAE along with any fixed expenses.

The basic premium is the (Expected Excess Loss & ALAE multiplied by the loss conversion factor) plus commission, general expenses, and UW profit.

B = $300,000 * (1 + 10.0%) + $55,000 + $15,000 + $5,000
= $405,000 **<= Basic Premium**

The tax multiplier, T, is 1 / (1 - 3.0%)

T = 1.031

To calculate the incurred retrospective rating premium we need L, the ratable loss amount.
We'll use the payment pattern to determine it at each point in time.
We're given the payment pattern, let's look at this information in more detail before working with it.

1. This is a 1-year incurred retrospective rating plan; no premium adjustments will occur until 18 months have elapsed, and then are evaluated annually.
2. The initial premium is paid immediately at the start, along with the commission.
3. We assume all losses are at ultimate after 7.5 years and that ALAE is included in the ratable loss.
4. Since it's a 1-year policy, all general expenses happen within the first year. ULAE is accrued all the time the losses aren't at ultimate.

Policyholder Cash Flow

Time (Years)	(7) Primary Incurred Loss & ALAE [Ratable Loss, L]	(8) Total Premium Paid	(9) Cumulative Cash Flow	(10) Incremental Cash Flow	Comments
0.00	0	$1,100,000	($1,100,000)	($1,100,000)	
0.25	$64,200	$1,100,000	($1,100,000)	$0	
0.50	$157,800	$1,100,000	($1,100,000)	$0	
0.75	$272,400	$1,100,000	($1,100,000)	$0	
1.00	$393,000	$1,100,000	($1,100,000)	$0	
1.50	$463,800	$943,551	($943,551)	$156,449	First premium adjustment occurs at t = 1.5. Losses better than expected so the policyholder receives a partial premium refund from the insurer.
2.50	$527,400	$1,015,679	($1,015,679)	($72,128)	
3.50	$563,400	$1,056,507	($1,056,507)	($40,828)	
4.50	$584,400	$1,080,323	($1,080,323)	($23,816)	Subsequent evaluations (t ≥ 2.5) show losses gradually deteriorating; this requires additional premium payments to the insurer.
5.50	$593,400	$1,090,530	($1,090,530)	($10,207)	
6.50	$598,200	$1,095,974	($1,095,974)	($5,444)	
7.50	$600,000	$1,098,015	($1,098,015)	($2,041)	

(7) = (1) * Expected Primary Loss & ALAE
(8) This is the initial premium until 1.5 years have elapsed. Afterwards, use (8) = [B + c * (7)] * T
(9) = -1 * (8) as these are the cumulative payments made by the policyholder.
(10) = [(9) current row] - [(9) prior row]

The ratable loss (column 7) is the primary incurred loss and ALAE after the consideration of any maximum or minimum ratable loss.

The requirement for the insured to make additional premium payments after the end of the policy period creates credit risk for the insurer.

Note:
In the text, Fisher uses an unrounded value of T. Here we've rounded T to 3 decimal places for convenience.

On the next page we look at the cash flow from the insurer's perspective.

Insurer Cash Flow

Columns (11) – (19) are cumulative figures to date

Time (Years)	(11) Premium	(12) Primary Loss & ALAE Paid	(13) Excess Loss & ALAE Paid	(14) Total Loss & ALAE Paid	(15) Total Commission	(16) Premium Tax	(17) General Expenses	(18) ULAE	(19) Cash Flow	(20) Incremental Cash Flow
0.00	$1,100,000	$0	$0	$0	$55,000	$33,000	$3,750	$0	$1,008,250	$1,008,250
0.25	$1,100,000	$12,600	$300	$12,900	$55,000	$33,000	$6,570	$6,570	$985,960	($22,290)
0.50	$1,100,000	$43,200	$1,500	$44,700	$55,000	$33,000	$9,375	$14,580	$943,345	($42,615)
0.75	$1,100,000	$87,000	$6,000	$93,000	$55,000	$33,000	$12,195	$23,850	$882,955	($60,390)
1.00	$1,100,000	$140,400	$15,000	$155,400	$55,000	$33,000	$15,000	$34,200	$807,400	($75,555)
1.50	$943,551	$245,400	$45,000	$290,400	$55,000	$28,307	$15,000	$44,280	$510,564	($296,836)
2.50	$1,015,679	$381,000	$105,000	$486,000	$55,000	$30,470	$15,000	$58,950	$370,259	($140,306)
3.50	$1,056,507	$478,800	$180,000	$658,800	$55,000	$31,695	$15,000	$71,910	$224,102	($146,157)
4.50	$1,080,323	$542,400	$240,000	$782,400	$55,000	$32,410	$15,000	$81,180	$114,333	($109,768)
5.50	$1,090,530	$573,600	$270,000	$843,600	$55,000	$32,716	$15,000	$85,770	$58,444	($55,889)
6.50	$1,095,974	$586,200	$285,000	$871,200	$55,000	$32,879	$15,000	$87,840	$34,055	($24,389)
7.50	$1,098,015	$600,000	$300,000	$900,000	$55,000	$32,940	$15,000	$90,000	$5,075	($28,980)

(11) = (8)
(12) = (2) * Expected Primary Loss & ALAE
(13) = (3) * Expected Excess Loss & ALAE
(14) = (12) + (13)
(15) Commission is paid upfront then doesn't change.

(16) = (11) * 3.0% (Premium tax rate)
(17) = (5) * General Expenses
(18) = (Expected Primary and Excess Loss & ALAE) * ULAE % * (6)
(19) = (11) - (14) - (15) - (16) - (17) - (18)
(20) = [(19) current row] - [(19) prior row]

Notes:

- Both the policyholder and insurer have negative cash flows after t = 2.5. The insurer has a negative cash flow because it is paying out on losses. The policyholder has a negative cash flow because they exchanged a larger upfront premium (guaranteed cost premium) for a lower initial premium with additional premium payments later and the potential to receive premium refunds if their experience was better than expected.
- The additional premium payments from the policyholder adjust but do not completely offset the loss experience paid by the insurer at the time.
- The final cash flow figure of $5,075 is a result of rounding the premium tax, T, to 3 decimal places. If the full precision is used then we are left with exactly the UW profit provision of $5,000.

Fisher_CashflowLDD (Problem 1)

Reading: Fisher.OtherLSPlans
Model: Source Text
Problem Type: Visualize the cash flow for both the policyholder and the insurer under a large dollar deductible rating plan.

Given

Pricing Assumptions	
$600,000	Expected Primary Loss & ALAE
$300,000	Expected Excess Loss & ALAE
$55,000	Commission
$15,000	General Expenses
$5,000	Underwriting Profit Provision
10.0%	ULAE
3.0%	Tax Rate

There is no aggregate excess loss exposure.

Payment Patterns

Time (Years)	Initial Premium	(1) Primary Incurred Loss & ALAE	(2) Primary Paid Loss & ALAE	(3) Excess Paid Loss & ALAE	(4) Total Paid Loss & ALAE	Commission	(5) General Expenses	(6) ULAE
0.00	100%					100%	25.0%	
0.25		10.7%	2.1%	0.1%	1.4%		43.8%	7.3%
0.50		26.3%	7.2%	0.5%	5.0%		62.5%	16.2%
0.75		45.4%	14.5%	2.0%	10.3%		81.3%	26.5%
1.00		65.5%	23.4%	5.0%	17.3%		100.0%	38.0%
1.50		77.3%	40.9%	15.0%	32.3%			49.2%
2.50		87.9%	63.5%	35.0%	54.0%			65.5%
3.50		93.9%	79.8%	60.0%	73.2%			79.9%
4.50		97.4%	90.4%	80.0%	86.9%			90.2%
5.50		98.9%	95.6%	90.0%	93.7%			95.3%
6.50		99.7%	97.7%	95.0%	96.8%			97.6%
7.50		100.0%	100.0%	100.0%	100.0%			100.0%

Find The premium charged for the large dollar deductible rating plan and illustrate the cash flows from both the policyholder and insurer perspectives.

Note As the CAS moves towards computer based testing, this type of problem (which is tedious to do by hand) becomes much easier to test.

Solution

Under a large dollar deductible rating plan the policyholder is responsible for all losses in the deductible layer, i.e. all of the primary losses.
However, since the insurer pays all claims and then seeks to recover the deductible layer from the policyholder, ULAE applies to all claims.

Using our Exam 5 knowledge the premium formula for a large dollar deductible rating plan is:
Premium = { Expected Excess Loss & ALAE + Commission + General Expense + UW Profit + [ULAE * (Expected Primary & Excess Loss & ALAE)] }* Tax Multiplier

The tax multiplier, T, is 1 / (1 - 3.0%)
T = 1.031

Also, unlike a retrospective policy, the premium charged for a large dollar deductible does not change over time. So we immediately get the LDD premium as:
Premium = { $300,000 + $55,000 + $15,000 + $5,000 + 10.0% * ($600,000 + $300,000) } * 1.031
= $479,381

We're given the payment pattern, let's look at this information in more detail before working with it.

1. This is a 1-year large dollar deductible rating plan and the insurer will recover losses in the deductible layer at the end of each quarter.
2. Since it's a 1-year policy, all general expenses happen within the first year. ULAE is accrued all the time the losses aren't at ultimate.
3. We assume all losses are at ultimate after 7.5 years.
4. Commission is paid in full immediately at policy inception.

Policyholder Cash Flow

Time (Years)	(7) Total Premium Paid	(8) Deductible Loss Reimbursements	(9) Cumulative Cash Flow	(10) Incremental Cash Flow
0.00	$479,381	$0	($479,381)	($479,381)
0.25	$479,381	$12,600	($491,981)	($12,600)
0.50	$479,381	$43,200	($522,581)	($30,600)
0.75	$479,381	$87,000	($566,381)	($43,800)
1.00	$479,381	$140,400	($619,781)	($53,400)
1.50	$479,381	$245,400	($724,781)	($105,000)
2.50	$479,381	$381,000	($860,381)	($135,600)
3.50	$479,381	$478,800	($958,181)	($97,800)
4.50	$479,381	$542,400	($1,021,781)	($63,600)
5.50	$479,381	$573,600	($1,052,981)	($31,200)
6.50	$479,381	$586,200	($1,065,581)	($12,600)
7.50	$479,381	$600,000	($1,079,381)	($13,800)

(7) Calculated at the top of the page.
(8) = (2) * Expected Primary Loss & ALAE
(9) = -1 * (7) - (8)
(10) = [(9) current row] - [(9) prior row]

The requirement for the insured to make additional payments for losses within the deductible layer creates credit risk for the insurer.

Note:

In the text, Fisher uses an unrounded value of T. Here we've rounded T to 3 decimal places for convenience.

On the next page we look at the cash flow from the insurer's perspective.

Insurer Cash Flow

Columns (11) – (19) are cumulative figures to date

Time (Years)	(11) Total Premium Received	(12) Deductible Loss Reimbursements	(13) Excess Loss & ALAE Paid	(14) Total Loss & ALAE Paid	(15) Total Commission	(16) Premium Tax	(17) General Expenses	(18) ULAE	(19) Cash Flow	(20) Incremental Cash Flow
0.00	$479,381	$0	$0	$0	$55,000	$14,381	$3,750	$0	$406,250	$406,250
0.25	$479,381	$12,600	$300	$12,900	$55,000	$14,381	$6,570	$6,570	$396,560	($9,690)
0.50	$479,381	$43,200	$1,500	$44,700	$55,000	$14,381	$9,375	$14,580	$384,545	($12,015)
0.75	$479,381	$87,000	$6,000	$93,000	$55,000	$14,381	$12,195	$23,850	$367,955	($16,590)
1.00	$479,381	$140,400	$15,000	$155,400	$55,000	$14,381	$15,000	$34,200	$345,800	($22,155)
1.50	$479,381	$245,400	$45,000	$290,400	$55,000	$14,381	$15,000	$44,280	$305,720	($40,080)
2.50	$479,381	$381,000	$105,000	$486,000	$55,000	$14,381	$15,000	$58,950	$231,050	($74,670)
3.50	$479,381	$478,800	$180,000	$658,800	$55,000	$14,381	$15,000	$71,910	$143,090	($87,960)
4.50	$479,381	$542,400	$240,000	$782,400	$55,000	$14,381	$15,000	$81,180	$73,820	($69,270)
5.50	$479,381	$573,600	$270,000	$843,600	$55,000	$14,381	$15,000	$85,770	$39,230	($34,590)
6.50	$479,381	$586,200	$285,000	$871,200	$55,000	$14,381	$15,000	$87,840	$22,160	($17,070)
7.50	$479,381	$600,000	$300,000	$900,000	$55,000	$14,381	$15,000	$90,000	$5,000	($17,160)

(11) Calculated at the top of the prior page.
(12) = (2) * Expected Primary Loss & ALAE
(13) = (3) * Expected Excess Loss & ALAE
(14) = (12) + (13)
(15) Commission is paid upfront then doesn't change.

(16) = (11) * 3.0% (Premium tax rate)
(17) = (5) * General Expenses
(18) = (Expected Primary and Excess Loss & ALAE) * ULAE % * (6)
(19) = (11) + (12) - (14) - (15) - (16) - (17) - (18)
(20) = [(19) current row] - [(19) prior row]

Notes:

- (12) Deductible Loss Reimbursements could also be called Primary Loss & ALAE Paid.
- The policyholder always has a negative cash flow (unless they experienced no claims in a quarter).
- The insurer has a negative cash flow after t = 0 because they have to pay general expenses during the first year plus ULAE on all claims, and pay out on the excess portion of any claims above the deductible.
- When all losses have reached ultimate and assuming all losses in the deductible layer are recovered, the insurer is left with the UW profit.

Reading:	Fisher.AggExcess	Fisher_AggDed1 (Problem 1)
Model:	Source text	
Problem Type:	Calculate the amount paid by the insured and the insurer under a per-occurrence and aggregate deductible policy.	

Given

A Commercial General Liability policy has a per-occurrence deductible and an aggregate deductible of:

$100,000 Per-occurrence deductible
$500,000 Aggregate deductible

Find

Calculate the insurance payments and the insured's cumulative deductible payment for each quarter.

Date	Dollars of loss on claims that are each less than $100,000 (1)	Number of claims over $100,000 (2)	Dollars of loss on claims over $100,000 (3)	Deductible (4)	Insurance payment (5)	Cumulative Deductible (6)
Q1	$132,500	0	$0	?	?	?
Q2	$93,000	2	$350,000	?	?	?
Q3	$105,000	0	$0	?	?	?
Q4	$122,500	1	$150,000	?	?	?

Solution

Each claim under the per-occurrence deductible is covered entirely by the insured unless their cumulative deductible payments reaches the aggregate limit. Each claim over the per-occurrence deductible contributes the per-occurrence limit to the cumulative deductible until the aggregate deductible limit is reached.

We're given ground-up losses for claims in excess of the per-occurrence limit. To solve the [problem, figure out the applicable deductible for each quarter. The insurance payment is the difference between the total claims and the deductible.

Applying this we get

Date	Dollars of loss on claims that are each less than $100,000 (1)	Number of claims over $100,000 (2)	Dollars of loss on claims over $100,000 (3)	Deductible (4)	Insurance payment (5)	Cumulative Deductible (6)
Q1	$132,500	0	$0	$132,500	$0	$132,500
Q2	$93,000	2	$350,000	$293,000	$350,000	$425,500
Q3	$105,000	0	$0	$74,500	$30,500	$500,000
Q4	$122,500	1	$150,000	$0	$372,500	$500,000

(4) = min{ (1) + (2) * [Per-occurrence limit], [Aggregate limit] - Prior row (6) }

(5) = (1) + (3) - (4)

Note:

A potential twist is being given aggregate losses in excess of the per-occurrence deductible in column (3) above.

Reading: Fisher.AggExcess

Fisher_AggDed2 (Problem 1)

Model: Source text

Problem Type: Calculate the amount paid by the insured and the insurer under a per-occurrence and aggregate deductible policy.

Given A Commercial General Liability policy has a per-occurrence deductible and an aggregate deductible of:

$100,000	Per-occurrence deductible
$500,000	Aggregate deductible

Find Calculate the insurance payments and the insured's cumulative deductible payment for each quarter.

Date	Dollars of loss on claims that are each less than $100,000 (1)	Number of claims over $100,000 (2)	Excess loss dollars on claims over $100,000 (3)	Deductible (4)	Insurance payment (5)	Cumulative Deductible (6)
Q1	$132,500	0	$0	?	?	?
Q2	$93,000	2	$350,000	?	?	?
Q3	$105,000	0	$0	?	?	?
Q4	$122,500	1	$150,000	?	?	?

Solution

Each claim under the per-occurrence deductible is covered entirely by the insured unless their cumulative deductible payments reaches the aggregate deductible. Each claim over the per-occurrence deductible contributes the per-occurrence limit to the cumulative deductible until the aggregate deductible is reached.

We're given excess losses for claims <u>over</u> the per-occurrence limit, i.e. we need to add the per-occurrence deductible back in to get the ground up loss. Now figure out the applicable deductible in each quarter. The insurance payment is the difference between the total claims and the deductible.

Applying this we get

Date	Dollars of loss on claims that are each less than $100,000 (1)	Number of claims over $100,000 (2)	Excess loss dollars on claims over $100,000 (3)	Deductible (4)	Insurance payment (5)	Cumulative Deductible (6)
Q1	$132,500	0	$0	$132,500	$0	$132,500
Q2	$93,000	2	$350,000	$293,000	$350,000	$425,500
Q3	$105,000	0	$0	$74,500	$30,500	$500,000
Q4	$122,500	1	$150,000	$0	$372,500	$500,000

(4) = min{ (1) + (2) * [Per-occurrence deductible], [Aggregate deductible] - Prior row (6) }
(5) = (1) + (2) * [per-occurrence deductible] + (3) - (4)

Reading:	Fisher.AggExcess	Fisher_UniTableM (Problem 1)
Model:	Source text	
Problem Type:	Calculate the Table M charge and insurance charge from first principles given a uniform aggregate loss distribution.	

Given	A ~ Uniform [0, 100]	Aggregate loss distibution
	50	Expected aggregate loss

Find Calculate the Table M Charge and insurance charge for the following actual losses.

A	Table M charge	Insurance Charge
40	?	?
50	?	?
60	?	?

Solution

Although the problem can be solved using a Lee diagram, we'll solve it with what we know from first principles.
By doing several problems like this you'll be able to apply the technique to any aggregate loss distribution the CAS may give you.

Insurance charge $= E \cdot \phi(r)$

Clearly, we need to form entry ratios. Remember, the (Table M) entry ratio is A / E, where A is the actual loss and E is the expected loss.

A	Entry Ratio
40	0.8
50	1.0
60	1.2

Table M Charge: $\phi(r) = \int_r^{\infty} (y - r)\mathrm{d}F(y)$

Alice: "Important detail: Let Y = A / E and let F be the cumulative distribution function of Y."

Since the expected loss is a scalar, namely 50, we scale the cumulative distribution for A by dividing it by this amount.
Since A is defined uniformly on [0, 100], this means Y = A / E is defined uniformly on [0, 2] (and is zero everywhere else).

Now we know the distribution for Y, we can write $F(y) = \frac{1}{2}y$ and so $\mathrm{d}F(y) = \frac{1}{2}\mathrm{d}y$

We'll illustrate plugging this into the Table M Charge formula for r = 0.8:

$$\phi(0.8) = \int_{0.8}^{2} (y - 0.8) \cdot \frac{1}{2}\mathrm{d}y = 0.36$$

The associated insurance charge at r = 0.8 is then $E \cdot \phi(0.8) =$ 50 * 0.36 = 18

Using the same process with the remaining entry ratios results in the following completed table.

A	Table M charge	Insurance Charge
40	0.36	18
50	0.25	12.5
60	0.16	8

Alice: "For those of you also reviewing the source, you'll see a similarity with Fisher's Chapter 3 Question 3. However, in the text (top p. 40) Fisher muddies the water by saying the insurance charge refers to an amount, not a ratio but then in Q3 asks the reader to find the ill-defined Table M insurance charge', which the solution shows is actually just the Table M Charge (i.e. a ratio), not the insurance charge."

Reading: Fisher.AggExcess

Model: Source text

Problem Type: Calculate the Table M insurance savings from first principles given an exponential aggregate loss distribution.

Given

A ~ Exponential Aggregate loss distribution
E[A] = 10

Find

Calculate the Table M Savings and insurance savings for the following actual losses.

A	Table M Savings	Insurance Savings
5	?	?
10	?	?
15	?	?

Solution

First we need the pdf and cdf for an exponential distribution with mean Θ

p.d.f. $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ c.d.f $F(x) = 1 - e^{-\frac{x}{\theta}}$

Next, we need the formula for the Table M Savings: $\psi(r) = \int_{0}^{r} (r - y) \mathrm{d}F(y)$

It's clear we're going to need to work with entry ratios instead of actual and expected losses.
Remember, the entry ratio is just the actual loss divided by the expected loss.

Form the new distribution: $Y = \frac{A}{E}$

Alice: "This is a really important part - forming the correct distribution."

We now need the p.d.f. and c.d.f. of the new distribution. To do this, it's helpful to recall the following relationship:

Let $Y = aX + b$ where a is non-zero. Then the p.d.f. of Y is given by $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$

Here $f_Y(y)$ is the p.d.f. of Y and $f_X(x)$ is the p.d.f. of X.

Using the above, our exponential distribution A which has mean 10, yields a = 1 / 10. So the p.d.f. of Y is:

$$f_Y(y) = \frac{1}{\left(\left|\frac{1}{10}\right|\right)} \cdot \frac{1}{10} e^{-\frac{\left(\frac{y}{\left(\frac{1}{10}\right)}\right)}{10}} = e^{-y}$$

This is an exponential distribution with mean 1 and so $F(y) = 1 - e^{-y}$.

Plugging this into the formula for the Table M Savings gives: $\psi(r) = \int_{0}^{r} (r - y) \cdot e^{-y} \mathrm{d}y$

By carefully evaluating this integral, we can complete the table as follows:

A	Entry Ratio	Table M Savings	Insurance Savings
5	0.5	0.1065	1.0653
10	1.0	0.3679	3.6788
15	1.5	0.7231	7.2313

Reading: Fisher.AggExcess
Model: Source text
Problem Type: Estimate the net insurance charge at an entry ratio.

Given

An insurer has five similar policies which have an aggregate limit and no per-occurrence limit.
Each policy has an expected loss of $150,000.

Claim #	Actual Loss
1	$132,000
2	$141,000
3	$150,000
4	$159,000
5	$168,000
Average	$150,000

Find

Using the above actual loss information, fill in the missing information below to calculate the net insurance charge at r = 0.6.

r	$\phi(r)$	$\psi(r)$
0.6	?	?

Solution

We're told there is no per-occurrence limit but there is an aggregate limit.
This means we would be dealing with a Table M although it is not necessary to know that to reach the answer.

First we calculate $\phi(r)$

This is the sum over all claims of the actual loss in excess of r*E, divided by the total expected loss.

$$\phi(r) = \frac{\sum_{Claims} \max(0, \text{Actual Loss}_i - r \cdot E)}{n \cdot E}$$

Here, n is the number of claims and E is the expected loss per claim.

= ($42,000 + $51,000 + $60,000 + $69,000 + $78,000) / (5 * $150,000)

= 0.4000 **<= This is the Table M charge**

The insurance charge is $E \cdot \phi(r) =$ $150,000 * 0.4000

= $60,000

Next, calculate $\psi(r)$

This is the sum over all claims smaller than r*E of the gap between r*E and the actual loss, divided by the total expected loss over all claims.

$\psi(r) =$ ($0 + $0 + $0 + $0 + $0) / (5 * $150,000)

= 0.0000 **<= This is the Table M savings**

The insurance saving is $E \cdot \psi(r) =$ $150,000 * 0.0000

= $0

Finally, the net insurance charge at r = 0.6 is the difference between the insurance charge and the insurance savings. That is

Net insurance charge = $60,000 - $0

= $60,000

Fisher_TblMBalEqDeriv

Reading: Fisher.Visualization
Model: Source text
Problem Type: Derive the Table M balance equations for a balanced retrospective rating plan

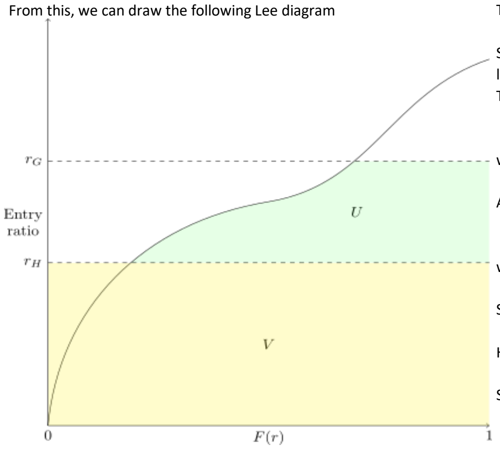
A retrospective rating plan has a basic premium, B, that is fixed (doesn't vary with loss). Assume the plan only has an aggregate limit/deductible.

We'll also assume our retrospective rating plan also has a minimum premium H and a maximum premium G.
These correspond to a minimum ratable loss L_H and a maximum ratable loss L_G respectively.

Using the retrospective rating formula, $R = (B + c \cdot L) \cdot T$ [1]
we get $H = (B + c \cdot L_H) \cdot T$ and $G = (B + c \cdot L_G) \cdot T$

Letting E[A] be the expected loss gives the following entry ratios: $r_H = \dfrac{L_H}{E[A]}$ and $r_G = \dfrac{L_G}{E[A]}$

From this, we can draw the following Lee diagram



The green and yellow area is the **<u>average ratable loss</u>**, so $\dfrac{E[L]}{E[A]} = U + V$ [2]

Since the premium for a retrospective rating plan should cover the expected loss, expenses, and taxes we have $R = (e + E[A]) \cdot T$
Taking the expectation of the retrospective rating formula, [1], and equating gives
$$(e + E[A]) \cdot T = (B + c \cdot (U + V) \cdot E[A]) \cdot T \quad [3]$$
where we used [2] to replace E[L]

At the minimum premium, H, the area of U is zero so we get
$$H = (B + cL_H) \cdot T = (B + cVE[A]) \cdot T$$

which rearranges to $\dfrac{H}{T} - B = cVE[A]$ [4]

Substituting [4] into [3] yields $(e + E[A]) \cdot T = cUE[A] \cdot T + H$ [5]

However, from the Lee diagram we know $U = \phi(r_H) - \phi(r_G)$ [6]

Substituting [6] into [5] yields
$$(e + E[A]) \cdot T = c \cdot \big(\phi(r_H) - \phi(r_G)\big) \cdot E[A] \cdot T + H$$

which gives the first balance equation: $\boldsymbol{\phi(r_H) - \phi(r_G) = \dfrac{(e + E[A]) \cdot T - H}{c \cdot E[A] \cdot T}}$

Next, ratable losses associated with the minimum premium may be expressed as $L_H = r_H \cdot E[A]$
Applying the retrospective rating formula yields $H = (B + c \cdot r_H \cdot E[A]) \cdot T$
Similarly we have $G = (B + c \cdot r_G \cdot E[A]) \cdot T$

Taking the difference of these two equations gives $G - H = c \cdot (r_G - r_H) \cdot E[A] \cdot T$

which rearranges to the second balance equation: $\boldsymbol{r_G - r_H = \dfrac{G - H}{c \cdot E[A] \cdot T}}$

<u>Note</u>:
The first balance equation tells us $\phi(r_H) - \phi(r_G)$
(the green shaded area labelled by U) is the difference between the expected retrospective premium at the minimum ratable loss and the minimum premium as R = (e + E[A])T, scaled by the factor cE[A]T.

Reading: Fisher.Visualization
Model: Source text
Problem Type: Derive the Limited Table M balance equations for a balanced retrospective rating plan

Fisher_LtdTblMBalEqDeriv

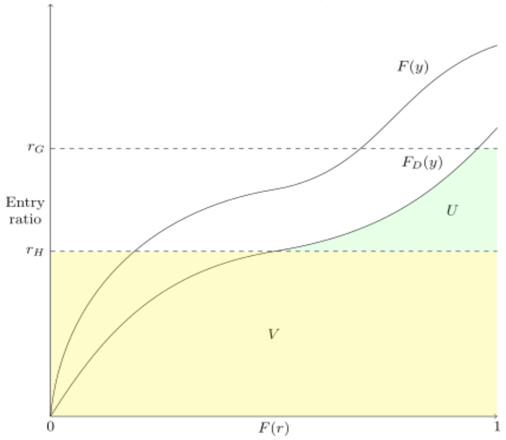
A retrospective rating plan has a basic premium, B, that is fixed (doesn't vary with loss). Assume the plan only has a per-occurrence limit/deductible.

We'll also assume our retrospective rating plan also has a minimum premium H and a maximum premium G.
These correspond to a minimum ratable loss L_H and a maximum ratable loss L_G respectively.

Using the retrospective rating formula, $R = (B + c \cdot L) \cdot T$ [1]
we get $H = (B + c \cdot L_H) \cdot T$ and $G = (B + c \cdot L_G) \cdot T$

Letting $E[A_D]$ be the expected limited loss gives the following entry ratios: $r_H = \dfrac{L_H}{E[A_D]}$ and $r_G = \dfrac{L_G}{E[A_D]}$

From this, we can draw the following Lee diagram



The green and yellow area is the **average ratable loss**, so $\dfrac{E[L]}{E[A_D]} = U + V$ [2]

Since the plan pays out on all losses, E[A], and we need to cover expenses and taxes we have $R = (e + E[A]) \cdot T$

Taking the expectation of the retrospective rating formula, [1], and equating gives

$$(e + E[A]) \cdot T = (B + c \cdot (U + V) \cdot E[A_D]) \cdot T \qquad [3]$$

where we used [2] to replace E[L]

At the minimum premium, H, the area of U is zero so we get

$$H = (B + cL_H) \cdot T = (B + cVE[A_D]) \cdot T$$

which rearranges to $\dfrac{H}{T} - B = cVE[A_D]$ [4]

Substituting [4] into [3] yields $(e + E[A]) \cdot T = cUE[A_D] \cdot T + H$ [5]

However, from the Lee diagram we know $U = \phi_D(r_H) - \phi_D(r_G)$ [6]

Substituting [6] into [5] yields

$$(e + E[A]) \cdot T = c \cdot \left(\phi_D(r_H) - \phi_D(r_G)\right) \cdot E[A_D] \cdot T + H$$

which gives the first balance equation: $\boldsymbol{\phi_D(r_H) - \phi_D(r_G) = \dfrac{(e + E[A]) \cdot T - H}{c \cdot E[A_D] \cdot T}}$

Next, ratable losses associated with the minimum premium may be expressed as $L_H = r_H \cdot E[A_D]$
Applying the retrospective rating formula yields $H = (B + c \cdot r_H \cdot E[A_D]) \cdot T$
Similarly we have $G = (B + c \cdot r_G \cdot E[A_D]) \cdot T$

Taking the difference of these two equations gives $G - H = c \cdot (r_G - r_H) \cdot E[A_D] \cdot T$

which rearranges to the second balance equation: $\boldsymbol{r_G - r_H = \dfrac{G - H}{c \cdot E[A_D] \cdot T}}$

Note:
The first balance equation tells us $\phi_D(r_H) - \phi_D(r_G)$
(the green shaded area labelled by U) is the difference between the expected retrospective premium at the minimum ratable loss and the minimum premium as R = (e + E[A])T, scaled by the factor $cE[A_D]T$.

Reading: Fisher.Visualization

Model: Source text

Problem Type: Derive the Table L balance equations for a balanced retrospective rating plan

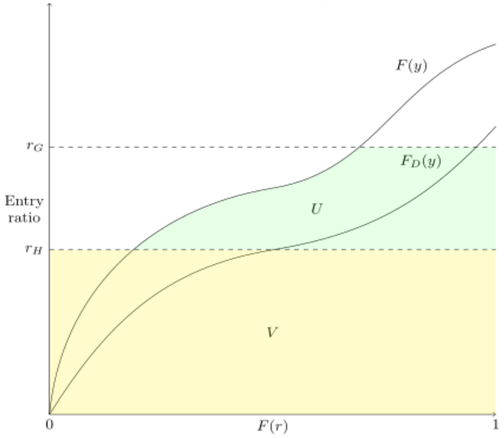
Fisher_TblLBalEqDeriv

A retrospective rating plan has a basic premium, B, that is fixed (doesn't vary with loss).
Assume the plan has both a per-occurrence limit/deductible and an aggregate limit/deductible.
We'll also assume our retrospective rating plan also has a minimum premium H and a maximum premium G.
These correspond to a minimum ratable loss L_H and a maximum ratable loss L_G respectively.

Using the retrospective rating formula, $R = (B + c \cdot L) \cdot T$ [1]
we get $H = (B + c \cdot L_H) \cdot T$ and $G = (B + c \cdot L_G) \cdot T$

Letting E[A] be the expected loss gives the following entry ratios: $r_H = \dfrac{L_H}{E[A]}$ and $r_G = \dfrac{L_G}{E[A]}$

From this, we can draw the following Lee diagram



The green and yellow area is the **average ratable loss**, so $\dfrac{E[L]}{E[A]} = U + V$ [2]

Since the plan pays out on all losses, E[A], and we need to cover expenses and taxes
we have $R = (e + E[A]) \cdot T$
Taking the expectation of the retrospective rating formula, [1], and equating gives

$$(e + E[A]) \cdot T = (B + c \cdot (U + V) \cdot E[A]) \cdot T \qquad [3]$$

where we used [2] to replace E[L]

At the minimum premium, H, the area of U is zero so we get

$$H = (B + cL_H) \cdot T = (B + cVE[A]) \cdot T$$

which rearranges to $\dfrac{H}{T} - B = cVE[A]$ [4]

Substituting [4] into [3] yields $(e + E[A]) \cdot T = cUE[A] \cdot T + H$ [5]

However, from the Lee diagram we know $U = \phi_D^\star(r_H) - \phi_D^\star(r_G)$ [6]

Substituting [6] into [5] yields

$$(e + E[A]) \cdot T = c \cdot \left(\phi_D^\star(r_H) - \phi_D^\star(r_G)\right) \cdot E[A] \cdot T + H$$

which gives the first balance equation: $\boldsymbol{\phi_D^\star(r_H) - \phi_D^\star(r_G) = \dfrac{(e + E[A]) \cdot T - H}{c \cdot E[A] \cdot T}}$

Next, ratable losses associated with the minimum premium may be expressed as $L_H = r_H \cdot E[A]$
Applying the retrospective rating formula yields $H = (B + c \cdot r_H \cdot E[A]) \cdot T$
Similarly we have $G = (B + c \cdot r_G \cdot E[A]) \cdot T$

Taking the difference of these two equations gives $G - H = c \cdot (r_G - r_H) \cdot E[A] \cdot T$

which rearranges to the second balance equation: $\boldsymbol{r_G - r_H = \dfrac{G - H}{c \cdot E[A] \cdot T}}$

Note:
The first balance equation tells us $\phi_D^\star(r_H) - \phi_D^\star(r_G)$
(the green shaded area labelled by U) is the difference between the expected retrospective premium at the minimum ratable loss and the minimum premium as R = (e + E[A])T, scaled by the factor cE[A]T.

Reading: Fisher.TableM
Model: Source Text
Problem Type: Calculate an empirical Table M using vertical slices

Given Experience for a group of risks with expected annual aggregate loss of $100,000.

Risk	Actual annual aggregate loss
1	20,000
2	50,000
3	60,000
4	70,000
5	80,000
6	80,000
7	90,000
8	100,000
9	150,000
10	300,000

Find Construct a Table M in increments of 0.1 from 0 to 3 using the vertical slicing method.

Table M: For Aggregate Expected Loss E = $100,000

r	ϕ(r)	φ(r)
0		
0.1		
0.2		

Solution

Fisher_Vert (Solution 1-1)

1.) Notice the risks in the group are already ordered by increasing actual loss. If your risks aren't ordered, do that first.

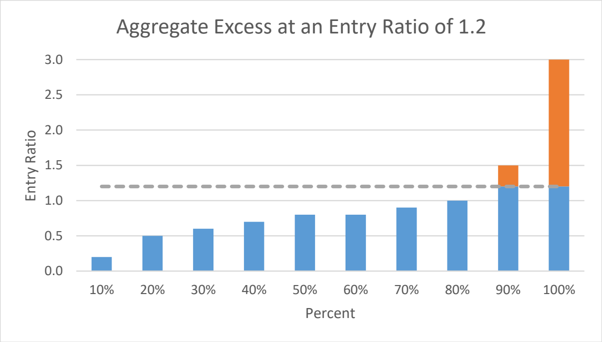
2.) Compute the entry ratio, r = [actual loss] / [expected loss], for each risk. Note they all have the same expected loss of $100,000

Note: If you were given the grouping by expected number of claims, use the average actual aggregate loss for the group as the expected aggregate loss.

Risk	Actual Loss	Entry Ratio
1	20000	0.2
2	50000	0.5
3	60000	0.6
4	70000	0.7
5	80000	0.8
6	80000	0.8
7	90000	0.9
8	100000	1.0
9	150000	1.5
10	300000	3.0

We want to build a Table M for entry ratios between 0 and 3 in increments of 0.1 using the vertical slicing method. We'll show the calculation for r = 1.2

3.) [Optional] Draw a Lee diagram and highlight the entry ratio being calculated.



In the graph, we've drawn a line corresponding to entry ratio r=1.2 and highlighted the portion of each risk's entry ratio that exceeds this.

4.) Calculate the portion of each entry ratio that exceeds the entry ratio under consideration and then the average value is the insurance charge, ϕ(r).

Risk	Actual Loss	Entry Ratio	Excess of r = 1.2
1	20000	0.2	0.0
2	50000	0.5	0.0
3	60000	0.6	0.0
4	70000	0.7	0.0
5	80000	0.8	0.0
6	80000	0.8	0.0
7	90000	0.9	0.0
8	100000	1	0.0
9	150000	1.5	0.3
10	300000	3	1.8
Total			2.10

Insurance charge at r = 1.2 is the Total / # risks = 2.1/10 = 0.21

5.) Compute the insurance savings using the formula: $\psi(r) = \phi(r) + r - 1$

$\psi(1.2) = \phi(1.2) + 1.2 - 1 = 0.21 + 0.2 = 0.41$

6.) Repeat this process for each entry ratio required in the Table M.

The next page shows the completed Table M, you should verify the calculation for a couple of the values.

Table M: For Expected Losses E = $100,000

r	ϕ(r)	φ(r)
0	1	0.00
0.1	0.90	0.00
0.2	0.80	0.00
0.3	0.71	0.01
0.4	0.62	0.02
0.5	0.53	0.03
0.6	0.45	0.05
0.7	0.38	0.08
0.8	0.32	0.12
0.9	0.28	0.18
1.0	0.25	0.25
1.1	0.23	0.33
1.2	0.21	0.41
1.3	0.19	0.49
1.4	0.17	0.57
1.5	0.15	0.65
1.6	0.14	0.74
1.7	0.13	0.83
1.8	0.12	0.92
1.9	0.11	1.01
2.0	0.10	1.10
2.1	0.09	1.19
2.2	0.08	1.28
2.3	0.07	1.37
2.4	0.06	1.46
2.5	0.05	1.55
2.6	0.04	1.64
2.7	0.03	1.73
2.8	0.02	1.82
2.9	0.01	1.91
3.0	0.00	2.00

Reading: Fisher.TableM
Model: Source Text
Problem Type: Calculate an empirical Table M using horizontal slices.

Given Experience for a group of risks with expected annual aggregate loss of $100,000

Risk	Actual annual aggregate loss
1	20,000
2	50,000
3	60,000
4	70,000
5	80,000
6	80,000
7	90,000
8	100,000
9	150,000
10	300,000

Find Construct a Table M using the horizontal slicing method.

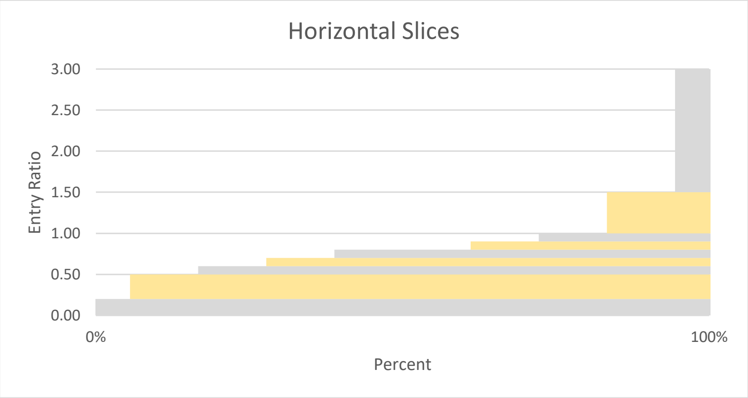
Solution

1.) Notice the risks in the group are already ordered by increasing actual loss. If your risks aren't ordered, do that first.

2.) Compute the entry ratio, r = [actual loss] / [expected loss], for each risk. Note they all have the same expected loss of $100,000.

Risk	Actual Loss	Entry Ratio, r
1	20000	0.2
2	50000	0.5
3	60000	0.6
4	70000	0.7
5	80000	0.8
6	80000	0.8
7	90000	0.9
8	100000	1.0
9	150000	1.5
10	300000	3.0

3.) [Optional] Draw a Lee diagram.



4.) For each distinct entry ratio, plus the 0 entry ratio, fill out the table below as follows:

a.) # Risks: This is the number of risks with entry ratio r

b.) # Risks over r: This is the number of risks with entry ratios strictly greater than r

c.) % Risks over r: This is b.) / [Total # of risks]

d.) Difference in r: This is the r value from the next row minus the r value from the current row. It is zero for the last row.

e.) Insurance charge: Start at the last row and work upwards. The last row always has zero insurance charge.
For row k, multiply the kth row difference in r by the kth row % risks over r then add this to the insurance charge for row k+1.

f.) Compute the insurance savings using the formula: $\psi(r) = \phi(r) + r - 1$

Entry Ratio, r	# Risks	# Risks over r	% Risks over r	Difference in r	ϕ(r)	φ(r)
0	0	10	100%	0.2	1.00	0.00
0.2	1	9	90%	0.3	0.80	0.00
0.5	1	8	80%	0.1	0.53	0.03
0.6	1	7	70%	0.1	0.45	0.05
0.7	1	6	60%	0.1	0.38	0.08
0.8	2	4	40%	0.1	0.32	0.12
0.9	1	3	30%	0.1	0.28	0.18
1.0	1	2	20%	0.5	0.25	0.25
1.5	1	1	10%	1.5	0.15	0.65
3.0	1	0	0%	0	0	2.00

Notice the horizontal method really only lends itself to calculating at entry ratios corresponding to known losses.
To calculate an "in-between" entry ratio insurance charge, form a trapezoid and add that area instead.
Fisher points out in practice there are usually sufficient losses to construct a Table M with intervals of 0.01 between rows and that linear interpolation is usually accurate enough.

Reading: Fisher.LimitedTableM
Model: Source Text Chapter 3 Q13
Problem Type: Calculate the total loss cost for the policy

Given The following is a table of insurance charges by per-occurrence deductible.

r	$10,000 Deductible	$20,000 Deductible
1.0	0.20	0.22
1.5	0.10	0.12
2.0	0.04	0.05
2.5	0.02	0.03

$40,000	Expected unlimited loss
$20,000	Expected primary loss at a per-occurrence limit of $10,000
$30,000	Expected primary loss at a per-occurrence limit of $20,000
$40,000	Aggregate deductible limit

Find

(a.) Suppose a policy has a $10,000 per-occurrence deductible. Calculate the following:
  i. The cost of the $40,000 aggregate deductible limit
  ii. The cost of the $10,000 per-occurrence deductible
  iii. The total cost of the policy

(b.) Suppose a policy has a $20,000 per-occurrence deductible. Calculate the following:
  i. The cost of the $40,000 aggregate deductible limit
  ii. The cost of the $20,000 per-occurrence deductible
  iii. The total cost of the policy

Solution

(a.) First we need to calculate the entry ratio(s) which characterises the policy. Here there's only one since there's an aggregate limit but no minimum.
The (characteristic) entry ratio is the ratio of the aggregate deductible limit to the expected primary loss at the per-occurrence limit.

Entry Ratio = $40,000 / $20,000 = 2

Now we can look up the insurance charge in the Limited Table M.
ϕ(2) = 0.04

i. The cost of the aggregate deductible limit is then: $E[A_D] \cdot \phi(r) =$ $20,000 * ϕ(2) = $800

ii. The cost of the per-occurrence deductible is: $E[A] - E[A_D] =$ $40,000 - $20,000 = $20,000

iii. The total cost of the policy is then: $20,000 + $800 = $20,800

(b.) We now repeat the process using the $20,000 per-occurrence deductible.
The (characteristic) entry ratio is: $40,000 / $30,000 = 1.3333

Now look up the insurance charge in the Limited Table M. We'll need to use linear interpolation between r = 1 and r = 1.5.

ϕ(1.3333) = 0.1533

i. The cost of the aggregate deductible limit is then: $E[A_D] \cdot \phi(r) =$ $30,000 * ϕ(1.3333) = $4,600

ii. The cost of the per-occurrence deductible is: $E[A] - E[A_D] =$ $40,000 - $30,000 = $10,000

iii. The total cost of the policy is then: $10,000 + $4600 = $14,600

Fisher_Ch3Q14 (Problem 1)

Reading: Fisher.TableL
Model: Source text: Chapter 3 Q14
Problem Type: Draw a Lee diagram and calculate the Table L insurance charge and savings

Given A policy has the following properties:

- Its unlimited loss distribution is continuous and uniform on the interval [0, 500]
- Its limited loss distribution is continuous and uniform on the interval [0, 400]
- Its entry ratio is 1.5 times the expected unlimited loss.

Find Draw a Lee diagram representing this policy and calculate the following:

a) $\phi(1.5)$

b) $\varphi(1.5)$

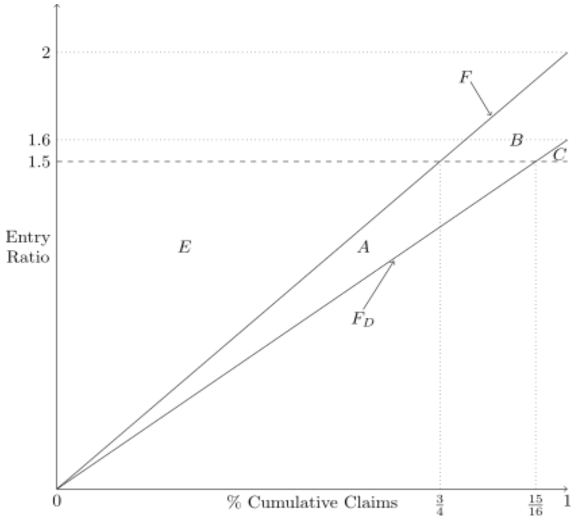
Solution Fisher_Ch3Q14 (Solution 1)

First we need to know the maximum entry ratio for the unlimited distribution. The unlimited loss distribution has an expected loss of 250
So its maximum entry ratio is 500 / 250 = 2.
Similiarly, we get the minimum entry ratio for the unlimited distribution as 0 / 250 = 0
Since the unlimited loss distribution is continuous and uniform, we know its Lee diagram will be a straight line from (0,0) to (1,2)

Next, we need to plot the limited loss distribution. Recall the formula for the Table L entry ratio is $\frac{\text{Limited Aggregate Loss}}{\text{Expected Unlimited Aggregate Loss}}$

Since the limited loss distribution is continuous and uniform, we know it will be represented by a straight line.
The minimum entry ratio for the limited loss distribution is 0 / 250 = 0
The maximum entry ratio for the limited loss distribution is 400 / 250 = 1.6

Bringing this all together yields the following Lee diagram



Alice: "On a side note there are two ways you can figure out the corresponding x coordinate for any given entry ratio.
First, you could find the equation of the line through (0,0) and (1,1.6) and then solve for x after substituting in the desired entry ratio for y.
The second way is to set the known entry ratio equal to the Table L entry ratio definition. Using an entry ratio of 1.5 as an example we have 1.5 = (Limited Actual Loss) / (Expected Unlimited Loss).
We know the expected unlimited loss is 250, so the limited actual loss must be 1.5 * 250 = 375.
Now, recalling we're interested in curve F_D, the maximum possible limited loss is 400. So the associated x value is 375/400 = 1.5/1.6."

From the Lee diagram we can deduce the areas which represent the Table L insurance charge and savings at an entry ratio of 1.5

Table L insurance charge = A + B + C
Table L insurance savings = A + E

Note the area under the curve F is equal to 1 and we can calculate x-axis coordinates by taking the ratio of the entry ratio to the maximum entry ratio for each curve (see Alice's sidenote). This gives

ϕ(1.5) = 1 - 0.5*1*1.6 + 0.5*(1 - 1.5 / 1.6) * (1.6 - 1.5) = 0.203125

Then using $\psi_D^* = \phi_D^*(r) + r - 1$

φ(1.5) = 0.203125 + 1.5 - 1 = 0.703125

Reading: Fisher.TableL

Model: Source Text

Problem Type: Construct a Table L from empirical data

Given Experience for a group of risks with a per-occurrence limit of $50,000

	Actual	
Risk	Unlimited Aggregate Loss	Limited Aggregate Loss
1	20,000	20,000
2	50,000	50,000
3	60,000	60,000
4	70,000	70,000
5	80,000	80,000
6	80,000	80,000
7	90,000	90,000
8	100,000	100,000
9	150,000	120,000
10	300,000	250,000
Average	100,000	92,000

Find Construct a Table L using the above data.

Solution

1.) Compute the excess ratio $k = \frac{E - E[A_D]}{E}$

Since we're not told the expected limited (or unlimited) aggregate losses, we approximate them with the average values from the table.
So the excess ratio is k = ($100,000 - $92,000) / $100,000 = 0.08

2.) Compute the entry ratio for each risk. Again, since the expected unlimited aggregate losses are unknown, use the average of all risks.

Remember: The Table L entry ratio is defined as $r = \frac{\text{Actual Limited Aggregate Loss}}{\text{Expected Unlimited Aggregate Loss}}$

Risk	Actual Unlimited Aggregate Loss	Actual Limited Aggregate Loss	Entry Ratio, r
1	20,000	20,000	0.20
2	50,000	50,000	0.50
3	60,000	60,000	0.60
4	70,000	70,000	0.70
5	80,000	80,000	0.80
6	80,000	80,000	0.80
7	90,000	90,000	0.90
8	100,000	100,000	1.00
9	150,000	120,000	1.20
10	300,000	250,000	2.50

3.) Apply the horizontal slicing method, making sure to arrange the unique entry ratios in **ascending** order and include a row for the 0 entry ratio.

Unique Entry Ratios	# Risks	# Risks over r	% Risks over r	Difference in r	$\phi_D^*(r) - k$	$\phi_D^*(r)$
0.00	0	10	100%	0.20	0.92	1.00
0.20	1	9	90%	0.30	0.72	0.80
0.50	1	8	80%	0.10	0.45	0.53
0.60	1	7	70%	0.10	0.37	0.45
0.70	1	6	60%	0.10	0.30	0.38
0.80	2	4	40%	0.10	0.24	0.32
0.90	1	3	30%	0.10	0.20	0.28
1.00	1	2	20%	0.20	0.17	0.25
1.20	1	1	10%	1.30	0.13	0.21
2.50	1	0	0%	0.00	0.00	0.08

The difference in r entry is the entry ratio in row k+1 minus the entry ratio in row k, where k is the current row. It is always 0 for the last row.
$\phi_D^*(r) - k$ is calculated as (% Risks over r) * (Difference in r) for row k, plus the entry for row k+1, column $\phi_D^*(r) - k$

4.) Complete the Table L by using the formula $\psi_D^*(r) = \phi_D^*(r) + r - 1$

Entry Ratio r	$\phi_D^*(r)$	$\psi_D^*(r)$
0.00	1.00	0.00
0.20	0.80	0.00
0.50	0.53	0.03
0.60	0.45	0.05
0.70	0.38	0.08
0.80	0.32	0.12
0.90	0.28	0.18
1.00	0.25	0.25
1.20	0.21	0.41
2.50	0.08	1.58

Reading: Fisher.TableL

Model: Source Text

Problem Type: Apply the ICRLL method to determine the total policy loss cost.

Fisher_ICRLLEx (Problem 1)

Given

Consider a workers' compensation policy which has the following characteristics:

$250,000	Per-occurrence limit
$750,000	Aggregate limit
$650,000	Expected unlimited aggregate loss
$490,000	Expected limited aggregate loss

The state/hazard group adjustment factor is 0.9

You may use the information in the following tables

Expected Loss Group	Range of Values
31	630,000 - 720,000
30	720,001 - 830,000
29	830,001 - 990,000
28	990,001 - 1,180,000
27	1,180,001 - 1,415,000
26	1,415,001 - 1,744,000

Table M	Expected Loss Group					
Entry Ratio	31	30	29	28	27	26
0.75	0.4150	0.4069	0.3989	0.3911	0.3833	0.3755
0.81	0.3864	0.3777	0.3690	0.3605	0.3521	0.3436
1.07	0.2867	0.2764	0.2661	0.2557	0.2453	0.2349
1.15	0.2628	0.2522	0.2417	0.2310	0.2203	0.2096
1.53	0.1797	0.1690	0.1583	0.1476	0.1369	0.1261

Find

Using the ICRLL method, calculate the total loss cost for the workers' compensation policy.

Solution

Since the ICRLL method is used to transform a Limited Table M into a Table M, we need to work with entry ratios from the Limted Table M at first.

1.) Compute the **Limited Table M** entry ratio $r = \frac{\text{Actual Limited Aggregate Loss}}{\text{Expected Limited Aggregate Loss}}$

Since the actual limited aggregate loss is (currently) unknown for the policy (we're pricing future losses), we substitute the aggregate policy limit for the actual limited aggregate loss.

This gives r = $750,000 / $490,000 = 1.53

2.) Compute the excess ratio $k = \frac{E - E[A_D]}{E}$

This gives k = (650,000 - 490,000) / 650,000 = 0.2462

3.) Compute the ICRLL adjustment $ICRLL = \frac{1 + 0.8k}{1 - k}$

This gives ICRLL = (1 + 0.8 * 0.2462) / (1 - 0.2462) = 1.5879

4.) Compute the adjusted expected loss = E * (State/hazard group adjustment) * ICRLL

This gives adjusted expected loss = $650,000 * 0.9 * 1.5879 = $928,921.50

5.) Find the expected loss group (ELG) that contains the adjusted expected loss.

This is ELG 29

6.) Look up ELG 29 and entry ratio 1.53 in the given Table M to get the insurance charge.

The insurance charge is 0.1583

7.) Calculate the aggregate limit charge, $\phi(r) \cdot E[A_D]$

This yields an aggregate limit charge of 0.1583 * $490,000 = $77,567

8.) Calculate the per-occurrence limit charge, $E - E[A_D]$

This yields a per-occurrence limit charge of $160,000

9.) Calculate the total loss cost of the policy = sum the per-occurrence and aggregate limit charges.

The total loss cost is $237,567

Reading: Bahnemann.Chapter5 Bahnemann_Ex5-4 (Problem 1)
Model: Source Text
Problem Type: Estimate the excess severity behaviour and determine the underlying distribution.

Given Grouped sample data for 1000 policies

Group	# Claims	Total Loss	Severity	$F_{1000}(x)$	$E_{1000}[\hat{X};x]$	$e_{1000}(x)$
0 - 100	100	6,000	60	0.100	96	1196
101 - 500	300	95,000	317	0.400	?	?
501 - 1000	240	145,000	604	0.640	606	1572
1001 - 2000	185	260,000	1405	?	?	?
2001 - 4000	140	450,000	3214	0.965	1096	2171
4001 - 5000	15	66,000	4400	?	?	?
5001 - 10000	20	150,000	7500	1.000	1172	NA
Total	1000	1,172,000	1172			

Find a.) Fill in the highlighted values and graph the excess severities to identify the distribution.

b.) Assuming a Pareto distribution is appropriate, calculate the parameters of the distribution.

Solution

a.)

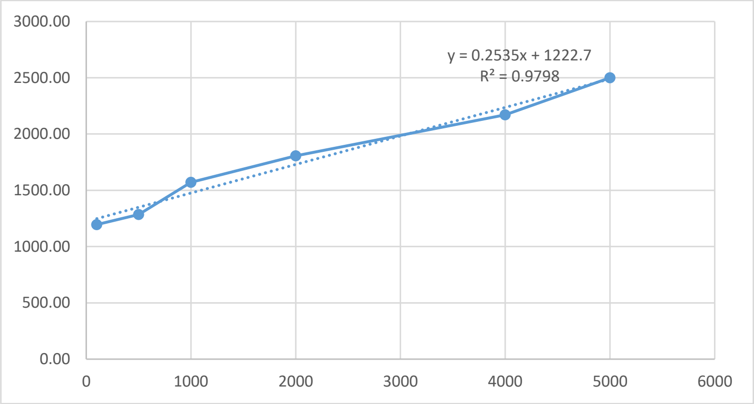
Group	# Claims	Total Loss	Severity	$F_{1000}(x)$	$E_{1000}[\hat{X}; x]$	$e_{1000}(x)$
0 - 100	100	6000	60	0.1	96	1195.56
101 - 500	300	95000	317	0.4	401	1285.00
501 - 1000	240	145000	604	0.64	606	1572.22
1001 - 2000	185	260000	1405	0.825	856	1805.714
2001 - 4000	140	450000	3214	0.965	1096	2171.429
4001 - 5000	15	66000	4400	0.980	1122	2500
5001 - 10000	20	150000	7500	1	1172	NA
Total	1000	1172000	1172			

The details below are for the 1001 - 2000 claim size group

$F_{1000}(2000)$ = (100 + 300 + 240 + 185) / 1000
= 0.825

$E_{1000}[\hat{X}; 2000]$= [(6000 + 95000 + 145000 + 260000) + (140 + 15 + 20)*2000] / 1000
= 856

$e_{1000}(2000)$ = (1172 - 856) / (1 - 0.825)
= 1805.71



b.) Since the graph of excess severities is very close to a straight line with positive slope, the underlying distribution is Pareto.

By equating the Pareto excess severity function with the equation for the linear regression trend line, we can estimate the parameters

$$e_{X(x)} = \frac{x+\beta}{\alpha-1} = 0.2535x + 1222.7$$

This gives $\frac{1}{\alpha-1} = 0.2535$ and $\frac{\beta}{\alpha-1} = 1222.7$

so $\alpha = 4.945$ and $\beta = 4823.27$

Reading: Bahnemann.Chapter6
Model: Source Text Example 6.3
Problem Type: Calculate ILFs loaded for expenses.

Given

Indemnity losses for a portfolio of insurance policies have a lognormal claim-size distribution with parameters

μ = 7
σ = 2.4

The policy per-claim limit applies only to the indemnity portion of a claim.

2,200	Average per-claim loss adjustment expense
0.0005	Claim frequency per exposure
35%	Variable expenses as a percentage of premium
100,000	Basic policy limit

L	E[X; L]
100,000	8,896.04
1,000,000	15,345.22

Find

a.) Calculate the increased limit factor for a policy limit of $1,000,000

b.) For a policy with 400 exposures, calculate the premium at the
i.) Basic limit
ii.) $1,000,000 limit.

c.) Suppose instead loss adjustment expenses are 20.0% of the indemnity portion of the claim. Calculate:
i.) The ILF for a policy with $1,000,000 limit.
ii.) Basic policy premium
iii.) Policy premium for a policy with $1,000,000 limit.

Useful Formulas

Lognormal Distribution

$$E[X] = e^{\mu+\frac{\sigma^2}{2}}$$

$$E[X;L] = \mathrm{E[X]}\cdot\Phi\left(\frac{\log(L)-\mu-\sigma^2}{\sigma}\right)+\mathrm{L}\cdot\Phi\left(\frac{-\log(L)+\mu}{\sigma}\right)$$

Solution

The expected value of a lognormal distribution and its limited expected value are given by:

$$E[X] = e^{\mu+\frac{\sigma^2}{2}} \text{ and } E[X;L] = \mathrm{E[X]} \cdot \Phi\left(\frac{\log(L)-\mu-\sigma^2}{\sigma}\right) + \mathrm{L} \cdot \Phi\left(\frac{-\log(L)+\mu}{\sigma}\right)$$

a.) Since we're given the dollar amount of the loss adjustment expenses we'll use the second formulation for the ILF

$$I(L) = \frac{E[X;L]+\epsilon}{E[X;b]+\epsilon}$$

E[X;100,000] = 8,896.04

E[X;1,000,000] = 15,345.22

ε = 2,200

So I($1,000,000) = (15345.22 + 2200) / (8896.04 + 2200)

= 1.5812

b i.) E[N] = m*ϕ = 400 * 0.0005 = 0.2

E[X;b] = 8896.04

Expected Loss Cost = mp = $E[N] \cdot (E[X;b] + \epsilon) =$ 2,219.21

Since we're not given any information about fixed expenses, we assume they are 0 and use a loss cost multiplier.

Basic Limit Premium = mp / (1 - v) = 2219.21 / (1 - 35.0%) = 3,414.17

b ii.) E[N] = 0.2 (from part b.i.)

E[X;L] = 15,345.22

Expected Loss Cost = mp = $E[N] \cdot (E[X;L] + \epsilon) =$ 3509.04

$1,000,000 Limit Premium = mp / (1 - v)

= 5,398.53

But it's much quicker to apply the ILF I($1 million) from part a.

$1,000,000 Limit Premium = $P_b \cdot I(L)$ (remember P_b is the basic limit premium)

= 3414.17 * 1.5812

= 5,398.49

(Minor differences due to rounding)

c i.) I($1,000,000) = $\frac{E[X;L]\cdot(1+\mu)}{E[X;b]\cdot(1+\mu)}$ (notice the ALAE expense cancels out)

= 1.7249

c ii.) E[N] = 0.2 (from part b.i.)

E[X;b] = 8,896.04

Expected Loss Cost = mp = $E[N] \cdot E[X;b] \cdot (1+\mu)$

= 2,135.05

Basic Premium = 3,284.69 (apply LCM as no fixed expenses)

c iii.) $1,000,000 Limit Premium = $P_b \cdot I(L)$

= 5,665.93

Reading: Bahnemann.Chapter6

Model: Source Text

Problem Type: Check increased limits factors for consistency

Bahnemann_Consistency (Problem 1)

Given

Per Occurrence Limit	Increased Limit Factor
$100,000	1.000
$200,000	1.240
$250,000	1.340
$500,000	1.515
$1,000,000	1.915

Find Determine if the ILFs satisfy the consistency test, and if not then identify the range of factors which would work.

Solution

We need to measure the incremental rate as coverage increases and then check that the amounts decrease

	Per Occurrence Limit, L	ILF	Marginal Rate per $1,000 Coverage
(a)	$100,000	1.000	NA
(b)	$200,000	1.240	0.0024
(c)	$250,000	1.340	0.0020
(d)	$500,000	1.515	0.0007
(e)	$1,000,000	1.915	0.0008

Sample calculation:

Marginal rate (c) = $(ILF_c - ILF_b) / [(L_c - L_b) / 1{,}000]$

The increased limit factors FAIL the consistency test because the marginal rate does not always decrease.

Observe the marginal rate for row (e) is greater than the marginal rate for row (d). This is why the test failed.
To correct it, we need the marginal rate for row (e) to be less than or equal to the marginal rate for row (d).
So we need

$(ILF_e - ILF_d) / [(L_e - L_d) / 1{,}000] \quad \leq 0.0007$

That is,

$ILF_e \leq 0.0007 * ((Le - Ld) / 1{,}000) + ILFd$

$= 1.865$

However, to avoid illogical rating, we also require $ILF_e \geq ILF_d$

So the acceptable range of values for ILF_e is

$1.515 < ILFe < 1.865$

Reading: Bahnemann.Chapter6
Model: Source Text
Problem Type: Calculate various aspects using a straight deductible

Given

Deductible, d	E[X;d]	$F_X(d)$	C(d)	(Modified) Frequency	Severity	Pure Premium
0	0	0	?	?	?	?
1,000	659	0.4847	?	?	?	?
2,000	1,111	0.5989	?	?	?	?
3,000	1,478	0.6625	?	?	?	?
5,000	2,071	0.7364	?	?	?	?
10,000	3,144	0.8215	?	?	?	?

0.0005	Ground-up claim frequency, ϕ
100,000	Basic policy limit
8,896	Basic limit expected loss, E[X;b]
0	Fixed ALAE per claim, ε
20%	Variable ALAE, u

Find

a.) Fill in the missing information in the table.

b.) For a policy with a deductible of $2,000
i.) Calculate the deductible-adjusted frequency
ii.) Calculate the modified severity.

c.) The basic limit premium for a policy is $3,285
The ILF for a $1,000,000 limit is 1.8074
Calculate the premium for a policy with $1,000,000 limit and $2,000 deductible.

Solution

Recall $C(d) = \frac{E[X;d] + F_X(d) \cdot \epsilon}{E[X;b] + \epsilon}$, modified frequency = $\phi(1 - F_X(d))$, and

modified severity = $\left(\frac{E[X;b] - E[X;d] + (1 - F_X(d))\epsilon}{1 - F_X(d)}\right) \cdot (1 + u)$

Notice if $\epsilon = 0$ then C(d) simplifies to $C(d) = \frac{E[X;d]}{E[X;b]}$

a.)

Deductible, d	E[X;d]	$F_X(d)$	C(d)	(Modified) Frequency	Severity	Pure Premium
0	0	0	0.0000	0.000500	$10,675	$5.34
1000	659	0.4847	0.0741	0.000258	$19,182	$4.94
2000	1111	0.5989	0.1249	0.000201	$23,291	$4.67
3000	1478	0.6625	0.1661	0.000169	$26,375	$4.45
5000	2071	0.7364	0.2328	0.000132	$31,070	$4.10
10000	3144	0.8215	0.3534	0.000089	$38,669	$3.45

b.)

i.) We can read this off directly from the frequency column for the $2,000 deductible row: 0.000201

ii.) Same row, severity column $23,291

c.) The formula is $P_b \cdot (I(L) - C(d))$

Here L = $1,000,000 and d = $2,000

P_L = $5,527.05

Reading: Bahnemann.Chapter6
Model: Source Text
Problem Type: Calculate modified severity and pure premium for a franchise deductible

Given

Deductible, d	E[X;d]	$F_X(d)$	C(d)	Frequency	Severity	Pure Premium
0	0	0	?	?	?	?
1,000	659	0.4847	?	?	?	?
2,000	1,111	0.5989	?	?	?	?
3,000	1,478	0.6625	?	?	?	?
5,000	2,071	0.7364	?	?	?	?
10,000	3,144	0.8215	?	?	?	?

0.0005	Ground-up claim frequency, ϕ
$8,896	Basic limit expected loss, E[X;b]
0	Fixed ALAE per claim, ε
20%	Variable ALAE, u

Find Complete the table.

Solution

For a franchise deductible we have: $C(d) = \dfrac{E[X;d] - d\big(1 - F_X(d)\big) + F_X(d)\epsilon}{E[X;b] + \epsilon}$

Changing the deductible type (but not the deductible amount) doesn't impact claim frequency, so the modifed claim frequency is still: $\phi(1 - F_X(d))$

However, the modified severity becomes: $\left(\dfrac{E[X;b] - E[X;d]}{1 - F_X(d)} + (d + \epsilon)\right)(1 + u)$ *Alice: "Note the + d in the large ( )"*

Using these equations we can complete the table as follows

Deductible, d	E[X;d]	$F_X(d)$	C(d)	Frequency	Severity	Pure Premium
0	0	0	0.00000	0.000500	$10,675	$5.34
1,000	659	0.4847	0.01615	0.000258	$20,382	$5.25
2,000	1,111	0.5989	0.03471	0.000201	$25,691	$5.15
3,000	1,478	0.6625	0.05233	0.000169	$29,975	$5.06
5,000	2,071	0.7364	0.08464	0.000132	$37,070	$4.89
10,000	3,144	0.8215	0.15277	0.000089	$50,669	$4.52

Reading: Bailey.Simon
Model: 2014.Q5
Problem Type: Experience of a single car-year

Given The following data shows the experience of a merit rating plan for a specific state

Number of Accident-Free Years	Earned Car Years	Earned Premium ($000)	Number of Incurred Claims
3 or more	250,000	250,000	1,200
2	300,000	100,000	625
1	25,000	100,000	750
0	12,000	150,000	1,500
Total	587,000	600,000	4,075

The base rate is $1,000 per exposure. No other rating variables are applicable.

Find

a.) The typical exposure base used to develop the merit rating plan is earned premium.
Briefly discuss two assumptions in selecting this exposure base.

b.) Calculate the ratio of credibility for an exposure with two or more years accident-free experience to one or more years accident-free experience.

c.) Calculate the premium for an exposure that is accident free for two or more years.

Solution

a.) 1.) High frequency territories must also be high premium territories.
2.) Territory differentials must be proper (adequate).

b.) Notice here we're not told the earned premium is at present rates or relative to a particular group.

This could mean that earned premium is not the most appropriate base to use.
However, we're told in part a.) the typical base is earned premium so we'll use earned premium.

First we need the frequency for the entire group. Then we'll compute the frequency and experience mod for each merit rating group.
Total Frequency = (Total Claims) / (Total Earned Premium)
= 4,075 / 600,000
= 0.006791667

Years Accident-Free	Frequency (1)	Experience Mod (2)	Credibility (3)
3 or more	0.0048	0.7067	0.2933
2 or more	0.0052	0.7677	0.2323
1 or more	0.0057	0.8425	0.1575

(1) = (Incurred Claims) / (Earned Premium)
(2) = (1) / (Total Frequency)
(3) Since each group has had zero accidents in at least the past year, we know R=0 and the credibility formula becomes Mod = 1 - Z.

The ratio of 2 or more to 1 or more year accident-free years credibility is: 1.4750

c.) From the table in part b.) above, the experience mod for the group with 2 or more years accident free is 0.7677. Then
Premium = (Base rate) * Mod
= 1,000 * 0.7677
= $767.75

Reading: Bailey.Simon
Model: 2012.Q6
Problem Type: Experience of a single car-year

Given An insurance company has a private passenger auto book of business with the following claims experience:

Territory	Years Since Last Accident	Earned Premium at Present Rates for Two Years Since Last Accident	Earned Car Years	Number of Claims	Incurred Loss
1	0	$15,000,000	15,000	5,000	$9,000,000
1	1	$125,000,000	125,000	41,000	$75,000,000
1	2+	$230,000,000	230,000	76,000	$138,000,000
2	0	$25,000,000	25,000	7,000	$16,000,000
2	1	$310,000,000	300,000	84,000	$187,000,000
2	2+	$550,000,000	535,000	147,000	$328,000,000
3	0	$10,000,000	10,000	4,000	$7,000,000
3	1	$80,000,000	100,000	35,000	$43,000,000
3	2+	$160,000,000	170,000	60,000	$100,000,000

Find Choose an appropriate exposure base for calculating credibility. Justify the selection.

Solution

There are two choices of exposure base which we could use: Earned Car Years or Earned Premium.
This question is testing the comments made by Hazam, that high frequency territories must be high premium territories and the differentials must be accurate.

We'll test the frequency requirement first.

Territory	(1) Earned Premium	(2) Earned Car Years	(3) Number of Claims	(4) Average Earned Premium	(5) Relative Earned Premium to Total	(6) Frequency	(7) Relative Frequency
1	$370,000,000	370,000	122,000	$1,000.00	1.003	0.330	1.085
2	$885,000,000	860,000	238,000	$1,029.07	1.032	0.277	0.910
3	$250,000,000	280,000	99,000	$892.86	0.896	0.354	1.163
Total	$1,505,000,000	1,510,000	459,000	$996.69	1.000	0.304	1.000

(4) = (1) / (2)
(5) = (4) / (Total 4)
(6) = (3) / (2)
(7) = (6) / (Total 6)

Observe Territory 2 has the highest earned premium relativity but the lowest frequency relativity. This contradicts Hazam's first point.

We now check to see if the territory differentials are appropriate.

Territory	Earned Premium	Incurred Loss	Loss Ratio
1	$370,000,000	$222,000,000	60.0%
2	$885,000,000	$531,000,000	60.0%
3	$250,000,000	$150,000,000	60.0%

Since all territories have the same loss ratio, the territory differentials are proper.
Thus, Hazam's second condition is satisfied.

Since Hazam's first condition is not met, it is more appropriate to use earned car years as the exposure base than earned premiums.

Reading: Bailey.Simon
Model: 2011.Q1
Problem Type: Credibility of a single car-year

Given

An insurance company is using a merit rating plan for drivers in two states.
State **X** has the following claims experience:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3 or more	$500,000	240
B	2	$150,000	125
C	1	$200,000	190
D	None	$300,000	300
Total		$1,150,000	855

State **Y** has the following relative claim frequencies for accident-free experience:

Number of Accident-Free Years	Relative Claim Frequencies to Total
3 or more	0.70
2 or more	0.77
1 or more	0.84

Find

Assuming no new risks enter or leave either state, use relative credibility to explain which state has more variation in an individual insured's probability of an accident.

Solution

We're given earned premiums **at present rates for group D** in State X.
This means we do not need to on-level the premiums or adjust them to account for the differentials between rating groups.

We're interested in the number of years claims-free which means we'll switch from groups A, B, C, and D to considering the sets A, A + B, A + B + C, and A + B + C + D.

First compute the **total claim frequency** for State X: = 855 / 1,150,000 = 0.000743

Then compute the **relative claim frequency** for each grouping of years accident-free in State X as follows:

Relative Claim Frequency = [(Number of Claims Incurred) / (Earned Premium at Present Group D Rates)] / (Total Claim Frequency)

Next, apply the experience mod formula: Mod = ZR + (1-Z)*1. Since we're dealing with past years accident-free, we know R = 0.
Recall the mod is the *Relative Claim Frequency,* so Z = 1 - Mod = 1 - *Relative Claim Frequency*

State X

Group	Number of Accident-Free Years	Relative Claim Frequencies to Total	Credibility	Re-base to 1 or more
A	3 or more	0.646	0.354	2.910
A + B	2 or more	0.755	0.245	2.010
A + B + C	1 or more	0.878	0.122	1.000

We're given most of the work already for State Y

State Y

Number of Accident-Free Years	Relative Claim Frequencies to Total	Credibility	Re-base to 1 or more
3 or more	0.700	0.300	1.875
2 or more	0.770	0.230	1.438
1 or more	0.840	0.160	1.000

By looking at the re-based columns for these two tables, we observe State X has ratios which are much closer to 1.0, 2.0, and 3.0.
This implies State X has more stable experience, and that State Y has more variation.

ISO_StandardCSLC (Problem 1)

Reading: ISO.Rating
Model: Source Text
Problem Type: Calculate the Company Subject Loss Cost using the standard approach

Given The following policy is being rated using the ISO CGL rating plan.

12/1/2014	Effective Date
Claims-Made (CM)	Policy Type
65%	Expected Loss Ratio (ELR)

	Annual Basic Limit Premium[1]
$75,000	Premises/Operations
$25,000	Products

Effective Date	Policy Type
12/1/2012	1st-year Claims-Made
12/1/2011	Occurrence
12/1/2010	Occurrence

Find Calculate the Company Subject Loss Cost using the standard approach using the information provided below.

[1] At $100,000 per-occurrence and actual aggregate limits.

Table 13B

Sub-line	Occurrence	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	1.20	1.32	1.62
Products	1.00	1.59	2.03	2.39

Table 13C

Sub-line	Occurrence	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	0.78	0.67	0.47
Products	1.00	0.43	0.35	0.22

Table 14

Year of Experience Period	Sub-line	Rule 5B	Rule 5C
Latest Year	Prem/Ops	0.907	0.926
	Products	0.882	0.901
2nd Latest Year	Prem/Ops	0.864	0.892
	Products	0.828	0.854
3rd Latest Year	Prem/Ops	0.823	0.858
	Products	0.777	0.810

Solution

First it's important to figure out the type of policy we're going to price. Since it's not stated in the question we need to apply our knowledge about the experience period. The experience period covers up to the latest three full policy years of experience and must end at least six months prior to the effective date. This means we can't use the policy effective 12/1/2013 because it's not complete, so we use the policies effective in 2010 – 2012.

Further, it's implicit that unless told otherwise, once you switch to a Claims-Made policy you remain on a Claims-Made policy. This means the policy effective 12/1/2013 would be a 2nd-year Claims-Made and so the policy being priced will be a **3rd year Claims-Made**.

Next, we need the Basic Limits Expected Loss for each sub-line.

This is the ELR multiplied by the annual basic limit premium where the per-occurrence limit is at the basic limit and the aggregate limit is the actual policy aggregate. We're given this information but watch out in the exam in case you need to apply an increased limit factor.

Prem/Ops BLEL = 65% * $75,000 = $48,750
Products BLEL = 65% * $25,000 = $16,250

We can now form the table used in the standard approach

(1) Policy Year	(2) Sub-line	(3) Policy Type	(4) BLEL	(5) PAF 13B	(6) PAF 13C	(7) De-trend	(8) Annual Basic Limits Company Loss Cost	
2012	Prem/Ops	1st-yr CM	$48,750	1.20	0.47	0.907	$24,938	
2012	Products	1st-yr CM	$16,250	1.59	0.22	0.882	$5,014	
2011	Prem/Ops	Occurrence	$48,750	1.20	1.00	0.864	$50,544	
2011	Products	Occurrence	$16,250	1.59	1.00	0.828	$21,393	
2010	Prem/Ops	Occurrence	$48,750	1.20	1.00	0.823	$48,146	
2010	Products	Occurrence	$16,250	1.59	1.00	0.777	$20,076	
							170,111	<= CSLC

Notes:

(5) We're pricing a **3rd-year Claims-Made** policy. PAF 13B always goes from the prospective policy to an occurrence policy so doesn't vary by year.
(6) PAF 13C translates from an occurrence policy to the historical policy so varies by policy year.
(7) We have no information to suggest there has been a dramatic change in exposures so apply Rule 5B from Table 14.
(8) = (4) * (5) * (6) * (7), Each row is rounded to nearest dollar.

ISO_PACR_CSLC (Problem 1)

Reading: ISO.Rating
Model: Source Text
Problem Type: Calculate the Company Subject Loss Cost using the Present Average Company Rate approach

Given The following policy is being rated using the ISO CGL rating plan.

12/1/2015	Effective Date
Claims-Made (CM)	Policy Type
70%	Expected Loss Ratio (ELR)
$100,000	Estimated Gross Annual Sales

	Annual Basic Limit Premium[1]
$80,000	Premises/Operations
$30,000	Products

Information about previous policy years

Effective Date	Policy Type	Gross Annual Sales
12/1/2014	3rd-year Claims-Made	$75,000
12/1/2013	2nd-year Claims-Made	$63,000
12/1/2012	1st-year Claims-Made	$42,000
12/1/2011	Occurrence	$29,000

Find Calculate the Company Subject Loss Cost using the Present Average Company Rate approach using the information provided below.

[1] At $100,000 per-occurrence and actual aggregate limits.

Table 13B

Sub-line	Occurrence	4th-yr CM	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	1.14	1.20	1.32	1.62
Products	1.00	1.51	1.59	2.03	2.39

Table 13C

Sub-line	Occurrence	4th-yr CM	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	0.84	0.78	0.67	0.47
Products	1.00	0.49	0.43	0.35	0.22

Table 14

Year of Experience Period	Sub-line	Rule 5B	Rule 5C
Latest Year	Prem/Ops	0.907	0.926
	Products	0.882	0.901
2nd Latest Year	Prem/Ops	0.864	0.892
	Products	0.828	0.854
3rd Latest Year	Prem/Ops	0.823	0.858
	Products	0.777	0.810

Solution

First it's important to figure out the type of policy we're going to price. Since it's not stated in the question we need to apply our knowledge about the experience period. The experience period covers up to the latest three full policy years of experience and must end at least six months prior to the effective date. This means we can't use the policy effective 12/1/2014 because it's not complete, so we use the policies effective in 2011 – 2013.

Further, it's implicit that unless told otherwise, once you switch to a Claims-Made policy you remain on a Claims-Made policy.
We're told the policy effective 12/1/2014 is a 3rd-year Claims-Made policy so the policy being rated is a **4th-year Claims-Made**.

Next, we need the Basic Limits Expected Loss for each sub-line. However, notice the rapid growth in annual sales. This suggests there has been a dramatic shift in exposures. Further, the estimated annual sales for the prospective period still shows growth.
This means we shouldn't use the standard approach and instead should use gross annual sales as the special exposure base.

The Basic Limits Expected Loss used in the present average company rate approach is the premium per prospective exposure multiplied by the historical exposure times the ELR.

(1) Policy Year	(2) Sub-line	(3) Policy Type	(4) Premium per Prospective Exposure	(5) Historical Exposure	(6) ELR	(7) BLEL	(8) PAF 13B	(9) PAF 13C	(10) Detrend	(11) Annual Basic Limits Company Loss Cost
2013	Prem/Ops	2nd-yr CM	0.80	$63,000	70%	35,280	1.14	0.67	0.926	24,953
2013	Products	2nd-yr CM	0.30	$63,000	70%	13,230	1.51	0.35	0.901	6,300
2012	Prem/Ops	1st-yr CM	0.80	$42,000	70%	23,520	1.14	0.47	0.892	11,241
2012	Products	1st-yr CM	0.30	$42,000	70%	8,820	1.51	0.22	0.854	2,502
2011	Prem/Ops	Occurrence	0.80	$29,000	70%	16,240	1.14	1.00	0.858	15,885
2011	Products	Occurrence	0.30	$29,000	70%	6,090	1.51	1.00	0.810	7,449
									CSLC =>	$68,330

Notes:

(4) Annual Basic Premium / Estimated Gross Annual Sales
(7) = (4) * (5) * (6)
(8) PAF 13B converts the prospective policy type to an occurrence policy, so doesn't vary by policy year.
(9) PAF 13C converts from an occurrence policy to the historical policy type.
(10) Since there is a dramatic shift in exposures we use Table 14 Rule 5C.
(11) = (7) * (8) * (9) * (10)

ISO_HistExp_CSLC (Problem 1)

Reading: ISO.Rating
Model: Source Text
Problem Type: Calculate the Company Subject Loss Cost using the Historical Exposures at Present Company Rates approach

Given The following policy is being rated using the ISO CGL rating plan.

12/1/2015	Effective Date
Claims-Made (CM)	Policy Type
75%	Expected Loss Ratio (ELR)

Current Company Rates per Exposure

Policy Type	Sub-line	Rate
3rd-yr CM	Prem/Ops	6.3
3rd-yr CM	Products	9.8
2nd-yr CM	Prem/Ops	4.9
2nd-yr CM	Products	5.9
1st-yr CM	Prem/Ops	2
1st-yr CM	Products	3.8
Ocurrence	Prem/Ops	9.3
Ocurrence	Products	8.5

Information about previous policy years

Effective Date	Policy Type	Policy Limits (occ/agg)	Gross Annual Sales
12/1/2014	3rd-year Claims-Made	250k/500k	$75,000
12/1/2013	2nd-year Claims-Made	150k/500k	$63,000
12/1/2012	1st-year Claims-Made	100k/250k	$42,000
12/1/2011	Occurrence	100k/200k	$29,000

Current Increased Limits Factors

Occurrence Limit	Aggregate Limit 200k	250k	500k
100k	1.000	1.240	1.570
150k	1.150	1.270	1.780
250k	1.330	1.450	1.920

Find Calculate the Company Subject Loss Cost using the Historical Exposures at Present Company Rates approach using the information provided below.

Table 13B

Sub-line	Occurrence	4th-yr CM	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	1.14	1.20	1.32	1.62
Products	1.00	1.51	1.59	2.03	2.39

Table 13C

Sub-line	Occurrence	4th-yr CM	3rd-yr CM	2nd-yr CM	1st-yr CM
Prem/Ops	1.00	0.84	0.78	0.67	0.47
Products	1.00	0.49	0.43	0.35	0.22

Table 14

Year of Experience Period	Sub-line	Rule 5B	Rule 5C
Latest Year	Prem/Ops	0.907	0.926
	Products	0.882	0.901
2nd Latest Year	Prem/Ops	0.864	0.892
	Products	0.828	0.854
3rd Latest Year	Prem/Ops	0.823	0.858
	Products	0.777	0.810

Solution

First it's important to figure out the type of policy we're going to price. Since it's not stated in the question we need to apply our knowledge about the experience period. The experience period covers up to the latest three full policy years of experience and must end at least six months prior to the effective date. This means we can't use the policy effective 12/1/2014 because it's not complete, so we use the policies effective in 2011 – 2013.

Further, it's implicit that unless told otherwise, once you switch to a Claims-Made policy you remain on a Claims-Made policy.
We're told the policy effective 12/1/2014 is a 3rd-year Claims-Made policy so the policy being rated is a **4th-year Claims-Made**.

Next, we need the Basic Limits Expected Loss for each sub-line. However, notice the rapid growth in annual sales. This suggests there has been a dramatic shift in exposures. This means we shouldn't use the standard approach and instead should use gross annual sales as the special exposure base. Since we're not given the prospective exposures but are given the full set of current rates this means we'll use the historical exposures at present company rates approach.

The Basic Limits Expected Loss used in the historical exposures at present company rates approach is the historical exposures multiplied by the basic rate per exposure then multiplied by an increased limit factor and the Expected Loss Ratio. From there we can form the following table:

(1) Policy Year	(2) Sub-line	(3) Policy Type	(4) Historical Exposures	(5) Basic Rate	(6) ILF	(7) ELR	(8) BLEL	(9) PAF 13B	(10) PAF 13C	(11) Detrend	(12) Annual Basic Limits Company Loss Cost
2013	Prem/Ops	2nd-yr CM	$63,000	4.9	1.570	75%	363,494	1.000	1.000	0.926	336,596
2013	Products	2nd-yr CM	$63,000	5.9	1.570	75%	437,677	1.000	1.000	0.901	394,347
2012	Prem/Ops	1st-yr CM	$42,000	2	1.240	75%	78,120	1.000	1.000	0.892	69,683
2012	Products	1st-yr CM	$42,000	3.8	1.240	75%	148,428	1.000	1.000	0.854	126,758
2011	Prem/Ops	Occurrence	$29,000	9.3	1.000	75%	202,275	1.000	1.000	0.858	173,552
2011	Products	Occurrence	$29,000	8.5	1.000	75%	184,875	1.000	1.000	0.810	149,749
										CSLC =>	1,250,684

Notes:

(5) This is the current basic limit rates for the company by policy type and sub-line.
(6) This is the ILF for the **basic per-occurrence limit** and the ***actual historical aggregate limit*** . Remember the basic limit is generally $100k.
(8) = (4) * (5) * (6) * (7)
(9) & (10) Since using today's rates there is no need to convert the type of policy.
(11) Since there is a dramatic shift in exposures we use Table 14 Rule 5C.
(12) = (8) * (9) * (10) * (11)

ISO_CalcExpMod (Problem 1)

Reading: ISO.Rating
Model: Source Text
Problem Type: Calculate the experience modification given the CSLC

Given The following policy is being rated using the ISO CGL rating plan.

12/1/2014	Effective Date
Claims-Made (CM)	Policy Type
65%	Expected Loss Ratio (ELR)
8/31/2014	Loss Evaluation Date

	Annual Basic Limit Premium[1]
$75,000	Premises/Operations
$25,000	Products

Policy Year	Sub-line	Annual Basic Limits Company Loss Cost
2012	Prem/Ops	24,938
2012	Products	5,014
2011	Prem/Ops	50,544
2011	Products	21,393
2010	Prem/Ops	48,146
2010	Products	20,076

Effective Date	Policy Type	Indemnity	ALAE
12/1/2012	1st-year Claims-Made	1,000	0
		2,200	0
		4,000	2,000
12/1/2011	Occurrence	0	3,000
		121,000	25,700
		5,000	102,000
12/1/2010	Occurrence	9,500	3,500
		5,500	0
		3,900	1,300
		2,800	0

Find Calculate the experience modification factor. You may use the information provided in the tables below.

Table 15 (exerpt)

	Latest Policy Year	Prior Policy Year	Next Prior Year
Sub-line	(18 Months)	(30 Months)	(42 Months)
Prem/Ops	0.536	0.337	0.183
Products	0.718	0.560	0.425
Sub-line	(21 Months)	(33 Months)	(45 Months)
Prem/Ops	0.483	0.287	0.151
Products	0.675	0.535	0.404

Development is measured from the policy effective date to the loss evaluation date.

Table 16 (exerpt)

CSLC	Credibility	EER	MSL
158,622 – 165,658	0.36	0.887	117,200
165,659 – 172,920	0.37	0.891	119,600
172,921 – 180,417	0.38	0.894	122,100

Solution

Applying our knowledge of the experience period we deduce the policy being rated is a **3rd-year Claims-Made**

Next, the CSLC is the sum of the Annual Basic Limits Company Loss Cost for the experience period.

CSLC = 170,111

We can use Rule 16 (Table 16) to find the credibility, Expected Experience Ratio (EER), and Maximum Single Loss (MSL).
By looking up the row which contains the CSLC of 170,111 we get:

0.37	Credibility (Z)
0.891	EER
119,600	MSL

Now we need to calculate the Actual Experience Ratio (AER). This is the sum of the expected future development and the limited claims history divided by the CSLC.

To calculate the **limited claims history**, notice we're given a list of indemnity and ALAE for the claims on each of the policies in the experience period.

We need calculate the **basic limits** indemnity and then cap the basic limits indemnity plus ALAE at the maximum single loss.

(1) Indemnity	(2) Basic Limits Indemnity	(3) ALAE	(4) Total	(5) Total Limited by MSL
1,000	1,000	0	1,000	1,000
2,200	2,200	0	2,200	2,200
4,000	4,000	2,000	6,000	6,000
0	0	3,000	3,000	3,000
121,000	100,000	25,700	125,700	119,600
5,000	5,000	102,000	107,000	107,000
9,500	9,500	3,500	13,000	13,000
5,500	5,500	0	5,500	5,500
3,900	3,900	1,300	5,200	5,200
2,800	2,800	0	2,800	2,800
				265,300

<= Limited claims history

<u>**Notes:**</u>

(2) = min((1), $100,000)

(4) = (2) + (3)

(5) = min((4), MSL)

Here, $100,000 is the basic per-occurrence limit.

The **expected future development** by policy type/sub-line is the product of the Annual Basic Limits Company Loss Cost multiplied by the EER and LDF.
We already found the EER and the LDFs are looked up in Rule 15 (Table 15).

Important point: LDFs only apply to <u>occurrence</u> policies!

Policy Year	Policy Type	Sub-line	(6) Annual Basic Limits Company Loss Cost	(7) EER	(8) LDF	(9) Expected Development
2012	1st-yr CM	Prem/Ops	24,938	0.891	0	0
2012	1st-yr CM	Products	5,014	0.891	0	0
2011	Occurrence	Prem/Ops	50,544	0.891	0.287	12,925
2011	Occurrence	Products	21,393	0.891	0.535	10,198
2010	Occurrence	Prem/Ops	48,146	0.891	0.151	6,478
2010	Occurrence	Products	20,076	0.891	0.404	7,227
				Expected Future Loss Development =>		36,828

(9) = (6) * (7) * (8), rounded to nearest dollar

From this, AER = (36,828 + 265,300) / 170,111 = 1.776

Mod = (AER - EER) / EER * Credibility
= (1.776 - 0.891) / 0.891 * 0.37
= 0.368

So the experience modification is a 36.8% debit.
and the experience modification factor equals 1.368

ISO_NoBasicPremium (Problem 1)

Reading: ISO.Rating
Model: Source Text
Problem Type: Calculate the basic limits expected loss cost when no basic premiums are available.

Given

A company has business in risk classes 2121 (Brewery) and 7390 (Beer & Ale Dealer – Wholesale & Drivers).
The brewery business generates the most premium dollars.

The company historically purchased a 150k/300k (per-occurrence/aggregate) Workers' Compensation policy.
The cost of this policy in the most recent experience year was $500,000.

Find

Using the information below, calculate the basic limits expected loss cost by sub-line (Prem/Ops and Products) for the company.

	Present Basic Limits Company Rate	
Sub-line	**2121**	**7390**
Prem/Ops	7.4	1.4
Products	5	2.9

65%	Company Expected Loss Ratio

Current Increased Limits Factors

Prem/Ops – 2121

		Aggregate Limits	
Occurrence Limit	200k	300k	500k
100k	1.000	2.024	5.664
150k	3.894	4.800	5.901
300k	3.975	5.264	5.953

Prem/Ops – 7390

		Aggregate Limits	
Occurrence Limit	200k	300k	500k
100k	1.000	2.774	4.263
150k	2.888	3.563	5.130
300k	4.992	5.761	6.154

Products – 2121

		Aggregate Limits	
Occurrence Limit	200k	300k	500k
100k	1.000	2.920	3.605
150k	3.610	4.762	5.407
300k	4.723	5.888	5.923

Products – 7390

		Aggregate Limits	
Occurrence Limit	200k	300k	500k
100k	1.000	1.316	4.663
150k	2.316	3.910	5.763
300k	3.469	4.764	6.509

Solution

The predominant risk class is 2121 (Brewery) as we're told this business generates the most premium dollars.
The annual company premium at the policy limits bought is $500,000

Sub-line	(1) Present Basic Limits Company Rate	(2) ILF @ Basic/Actual Policy Limits (per-occ / agg)	(3) ILF @ Policy Limits Purchased	(4) Present Basic Limits Company Rate	(5) Present Rate At Limits Bought
Prem/Ops	7.4	2.024	4.800	14.978	35.520
Products	5	2.920	4.762	14.600	23.810
Total				29.578	59.330

Sub-line	(6) Annual Basic Limits Company Premium	(7) Sub-line Weight	(8) Sub-line Annual Basic per-occ @ Actual Agg Limits	(9) ELR	(10) Basic Limits Expected Loss Cost
Prem/Ops	249,263	0.506	126,223	65%	82,045
Products	249,263	0.494	123,041	65%	79,976

Notes:

All factors and rates used are for risk class 2121 as this is the predominant class.
No rounding is used until step 10 where rounding occurs to the nearest dollar.

(2) Recall the basic per-occurrence policy limit for Prem/Ops and Products is 100,000. Use the actual aggregate limit purchased.
(3) ILF using the actual per-occurrence and actual aggregate limits purchased.
(4) = (1) * (2)
[Total 4] = sum (4)
(5) = (1) * (3)
[Total 5] = sum (5)
(6) = (Annual company premium at actual limits bought) * [Total 4] / [Total 5]
(7) = (4) / [Total 4]
(8) = (6) * (7)
(10) = (8) * (9)

If the basic per-occurrence limit/actual aggregate limit increased limits factor was not available then use the ILF available which has the closest aggregate limit to the purchased aggregate limit.

Q5_2015 (Problem 1)

Reading: Couret.Venter
Model: 2015.Q5
Problem Type: Calculate Multi-Dimensional Credibility

Given

An actuary estimated the loss cost for workers compensation insurance using a multi-dimensional credibility method.

Given the following:

- There were 2 classes in Hazard Group X.
- There were no major or minor permanent partial losses.
- Premium information was not available.
- Holdout sample of odd years was used as a proxy of the true mean.

Claim Count by Injury Type for Hazard Group X

	Even Year 1			Even Year 2		
	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)
Class 1	2	10	1,000	1	12	1,000
Class 2	3	10	1,000	2	13	1,000
Total	5	20	2000	3	25	2000

Find

Optimal Weights for Estimation of Permanent Total Injury Ratio

Fatal	Permanent Total
0.2	0.3

a.) Determine the ratio of permanent total injury to temporary total injury for Class 2 using a multi-dimensional credibility method.

b.) Fully describe the steps involved in performing a quintile test to evaluate the actuary's work.

c.) Briefly describe one shortcoming of the individual class sum of squared errors test and briefly describe why the quintiles test is a better way to evaluate the actuary's work.

Solution

a.) Since we're told there are no Major or Minor injuries, we only have F, PT, and TT to work with. This means the equation we need is:

$$E[w_2] = E[W] + b_w * (V_2 - E[V]) + c_w * (W_2 - E[W])$$

Here, we're using V for Fatal claims and W for PT claims. There is no variable for TT because we are calculating relativities to TT claims.
The subscript 2 is used because we want the credibility for class 2 using information about the Hazard Group F and PT claims.

Notice we're given the "optimal weights". This means we're given the credibilities produced by the multi-dimensional credibility technique.
That is, we know $b_w = 0.2$ and $c_w = 0.3$

We calculate E[V] and E[W] using both years of data (told both are even years) from the hazard group, i.e. across all classes in the hazard group.

$$E[V] = \frac{5+3}{2000+2000}$$

$$E[W] = \frac{20+25}{2000+2000}$$

We repeat this to get V_2 and W_2
Where this time we only use the information from class 2 for both years.

$$V_2 = \frac{3+2}{1000+1000}$$

$$W_2 = \frac{10+13}{1000+1000}$$

Substituting all of the above into the first equation gives the answer

$$E[w_2] = 0.011425$$

b.) Calculate ratios for all classes using the multi-dimensional credibility technique for all classes in the training set.
Rank the classes from smallest to largest by credibility relativity. Group into five quintiles and
calculate the relativity of the quintile ratio to the hazard group ratio for the following 3 predictions:
multi-dimensional credibility process, raw data, hazard group relativity.
Calculate the sum of squared errors for each of the 3 against the holdout data.
The method with the lowest sum of squared errors is the best.

c.) There is too much noise in the individual test. Grouping into quintiles reduces class specific variation
This gives more credible results, allowing us to assess the effectiveness of the credibility method.

Reading: Couret.Venter
Model: 2011.Q2
Problem Type: Multi-Dimensional Credibility

Given A multi-dimensional credibility technique has been developed to predict claim frequencies for major permanent partial claims.

- Seven years of data were collected.
- The technique produced a raw predicted relativity based on the oldest five years.
- The most recent two years were used as the holdout sample.

Quintile	Holdout Sample Relativity	Prediction Based on Raw	Prediction based on Credibility Procedure
1	0.6	0.3	0.4
2	0.8	0.5	0.7
3	1.0	1.1	1.0
4	1.2	1.9	1.5
5	1.4	3.0	1.8

Find Demonstrate whether the credibility technique produces an improved estimate using the sum of squared errors.

Solution

This question is fairly straightforward. It requires you to recall how Couret & Venter calculated the various sum of squared errors.

Couret & Venter considered three approaches:
1. Predicting based on the total hazard group relativity (which is always 1.0 when the sample means are normalized within a hazard group)
2. Predicting based on the raw data from the training data set.
3. Predicting using the multi-dimensional credibility procedure.

In each case, the predictions are compared against the holdout data set.

1. Sum of squared errors = (1 - 0.6)^2 + (1 - 0.8)^2 + (1 - 1)^2 + (1 - 1.2)^2 + (1 - 1.4)^2
= 0.4

2. Sum of squared errors = (0.3 - 0.6)^2 + (0.5 - 0.8)^2 + (1.1 - 1)^2 + (1.9 - 1.2)^2 + (3 - 1.4)^2
= 3.24

3. Sum of squared errors = (0.4 - 0.6)^2 + (0.7 - 0.8)^2 + (1 - 1)^2 + (1.5 - 1.2)^2 + (1.8 - 1.4)^2
= 0.3

Since the third method produces the lowest sum of squared errors, the multi-dimensional credibility procedure is an improvement over the hazard group membership method and the method which uses the raw data.

Reading:	Couret.Venter	Q5_2012 (Problem 1)
Model:	2012.Q5	
Problem Type:	Multi-Dimensional Credibility	

Given

The following data is used to price an excess of loss workers compensation policy:

- Data is available for the following injury types: Fatal, Permanent Total injury (PT), Major permanent partial (Major), minor permanent partial (Minor), temporary total (TT), and medical-only (Med).
- A multi-dimensional credibility technique (predicted) was used to estimate the frequency for class 5160.
- Class 5160 is in hazard group F.

Hazard Group F						
	Fatal	PT	Major	Minor	TT	Med
Frequency Relativity to TT	0.006	0.006	0.085	0.37	1.00	3.6
Severity Relativity to TT	80	100	30	4	1.00	0.3
Loss Elimination Ratio at $250,000	27%	22%	57%	100%	100%	100%

TT Frequency per $100 payroll	0.0002
TT Severity for Hazard Group F	$10,000

Hazard Group F for Fatal Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.75	0.70	0.90
Quintile 2	0.90	0.90	0.95
Quintile 3	1.00	1.00	1.00
Quintile 4	1.10	1.10	1.05
Quintile 5	1.25	1.30	1.10
Mean	1.00	1.00	1.00

Hazard Group F for PT Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.70	0.75	0.80
Quintile 2	0.90	0.90	0.90
Quintile 3	1.00	1.00	1.00
Quintile 4	1.15	1.20	1.10
Quintile 5	1.20	1.25	1.20
Mean	1.00	1.00	1.00

- The hazard group relativities for Major, Minor, TT, and Med will be used.
- The multi-dimensional credibility relativities for PT claims will be used.
- Class 5160 is in Quintile 4 for both Fatal and PT claims.

Find

a.) Determine whether multi-dimensional credibility relativities should be used to estimate the expected loss for fatal claims.

b.) Based on part a. above, calculate the expected loss for an excess of $250,000 workers compensation policy with $10 million in payroll.

Solution

This is a huge amount of information but the questions aren't actually that bad.

a.) We need to calculate the sum of squared errors for each of the credibility technique and the hazard group technique.
The data is already split into quintiles for us, and the overall means are 1.00 so no normalization is required.
We're told we only need to worry about the Fatal claims.

Multi-Dimensional Credibility Technique SSE (Predicted to Holdout)

SSE = (0.75 - 0.9)^2 + (0.9 - 0.95)^2 + (1 - 1)^2 + (1.1 - 1.05)^2 + (1.25 - 1.1)^2
= 0.050

Hazard Group SSE (Hazard group to Holdout)

SSE = (1 - 0.9)^2 + (1 - 0.95)^2 + (1 - 1)^2 + (1 - 1.05)^2 + (1 - 1.1)^2
= 0.025

Since the sum of squared errors is lower for the hazard group method, we won't use the multi-dimensional credibility relativities for fatal claims.

Note that for the hazard group, we used the mean of the entire hazard group **after normalization** which is 1.

b.) This part of the question is more like an IQ type of question. It blends knowledge from several papers.
From the first table we know all Minor, TT and Med losses are eliminated with a $250,000 attachment point.
This means we only need to look at Fatal, PT, and Major claims.

By our answer to part a. we'll use the fatal hazard group relativities, and in the question we're told to use the Major hazard group relativities.
A claim can't be both Fatal and Major so we calculate these separately. For PT claims we're told to use the credibility method relativities.
A key part of the problem is understanding how to relate the hazard group relativities to those produced by the credibility method or raw method.

To use the multi-dimensional credibilities or the raw credibilities, multiply the hazard group relativities by the quintile relativity which contains the class being priced.
If you're only using the hazard group relativities then you just multiply the hazard group relativities by 1.

Fatal relative to TT

Pure premium relativity = 0.48 (Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER = 0.3504 (Frequency x Severity) x (1 - LER)
Relative to hazard group = 0.3504 (Frequency x Severity) x (1 - LER) x 1.000

Major relative to TT

Pure premium relativity = 2.55 (Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER = 1.0965 (Frequency x Severity) x (1 - LER)
Relative to hazard group = 1.0965 (Frequency x Severity) x (1 - LER) x 1.000

PT relative to TT

This is more complicated since we're told to use the multi-dimensional credibility relativities.
We're told Class 5160 is in Quintile 4.
Using the Hazard Group F for PT Claims table, we look up the predicted value for quintile 4 which is 1.15.
This is the multi-dimensional credibility relativity for the class.

Pure premium relativity = 0.60 (Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER = 0.468 (Frequency x Severity) x (1 - LER)
Credibility Relativity = 1.15
Relative to hazard group = 0.5382 (Frequency x Severity) x (1 - LER) x 1.15

Summing the results gives the overall relativity to TT = 1.9851
TT pure premium per $100 of payroll = $2 By multiplying the frequency and severity in the second table.

We're told the company has $10 million in payroll. We convert this into $100s of payroll, multiply it by the TT pure premium and then multiply by the overall relativity to TT.
Answer to b.) $397,020

NCCI_ExperienceEx (Problem 1)

Reading: NCCI.ExperienceRating
Model: 2016.Q10
Problem Type: Calculate the experience rating modification

Given

An insured is subject to experience rating under the National Council on Compensation Insurance (NCCI)'s Experience Rating Plan Manual for Workers' Compensation and Employers Liability Insurance. The following information about the insured is given:

Payroll (Experience Period)	$5,000,000
State	AL
Class	7705

The following claims apply to the experience period. Each claim involves only one person, and none are disease claims:

Claim Number	Type	Loss
1	Indemnity	$29,000
2	Medical	$30,500
3	Indemnity	$90,000
4	Indemnity	$1,500
5	Medical	$45,000

Find

Calculate the experience modification. You may use the information provided below.

Class Code	ELR	D Ratio
7705	2.02	0.17
7710	1.41	0.13

AL Primary/Excess Split Point
$5,250

Expected Losses	Weighting Value
92,134 – 106,385	0.14
106,386 – 120,906	0.15

Expected Losses	Ballast Value
95,999 – 128,908	28,000
128,909 – 162,618	31,500

G	7
State Per Claim Accident Limit	$175,500
State Multiple Claim Accident Limit	$351,000

Solution

Using the first table provided we can look up the ELR and discount ratio for the risk. It's implicit this is an intrastate risk for Alabama.

ELR: 2.02

Discount Ratio: 0.17

Next, we need the expected losses. Find this by multiplying the ELR by the payroll divided by 100.

Expected Loss = 2.02 * $5,000,000 / $100

= $101,000 **<= Expected Loss, E**

Using the discount ratio we can split this into the expected primary and expected excess losses.

Expected Primary Loss = Discount Ratio * Expected Loss

= $17,170 **<=$E_p$**

Expected Excess Loss = Expected Loss - Expected Primary Loss

= $83,830 **<= E_e**

Now we can look up the weighting value and ballast value based on the expected loss.

Weighting Value = 0.14 **<= W**

Ballast Value = 28,000 **<= B**

All that remains now is to calculate the actual primary and actual excess losses subject to the state accident limits.
Since each claim only involves one person, the state multiple claim accident limit doesn't apply.
Since no claims are over the state per claim accident limit, no individual loss capping is required either.

			(1)	(2)
Claim Number	Type	Loss	Primary Loss	Excess Loss
1	Indemnity	29,000	5,250	23,750
2	Medical	30,500	1,575	7,575
3	Indemnity	90,000	5,250	84,750
4	Indemnity	1,500	1,500	0
5	Medical	45,000	1,575	11,925
Total		A_p =>	15,150	128,000

<= A_e

Notes:
(1) = min($5,250; Loss). The result is reduced by 70% if Loss Type = Medical
(2) = Loss - (1), if Loss Type = Medical then Loss is reduced by 70%

Now apply the experience modification formula:

$$\text{Experience Modification} = \frac{A_p + (1 - W) \cdot E_e + B + W \cdot A_e}{E_p + (1 - W) \cdot E_e + B + W \cdot E_e}$$

Experience Mod = (15,150 + (1 - 0.14)*83,830 + 28,000 + 0.14*128,000) / (17,170 + (1 - 0.14)*83,830 + 28,000 + 0.14*83,830)

= 1.03 **<= Note rounded to 2 decimal places**

Now check the maximum debit criteria:

$$\text{Max. Debit Mod} = 1.10 + 0.0004 \cdot \left(\frac{\text{Expected Loss}}{G}\right)$$

Max Debit Mod = 1.10 + 0.0004 * 101,000 / 7)

= 6.87

Experience Mod = min(1.03,6.87)

= 1.03 **<= Final answer**

Reading: Mahler.Credibility
Model: 2018.Q1a
Problem Type: Apply Chi-squared testing

Given

An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
Accident Years	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	12,033,000	1,766,000	2,329,000	1,236,000	170	170
1996-2000	13,812,000	1,819,000	2,762,000	1,273,000	210	172
2001-2005	13,985,000	1,751,000	2,797,000	1,506,000	210	201
2006-2010	16,444,000	1,710,000	3,288,000	1,471,000	240	195
2011-2015	17,507,000	1,673,000	3,350,000	1,439,000	250	198
Total	73,781,000	8,719,000	14,526,000	6,925,000	1,080	936

Find

a) Conduct chi-squared tests with an α value of 0.10 on actual vs. expected claims counts to confirm whether or not risk parameters have shifted over time.

Use the following table of critical values:

Degrees of Freedom	Critical Value (α = 0.10)
1	2.706
2	4.605
3	6.251
4	7.779
5	9.236
6	10.645

Solution

a) This is part of an integrative question (IQ). As such, it's worth looking harder for potential wrinkles.
Looking at the ultimate claim counts it is clear both lines of business have experienced an increase in claims.
However, looking at the earned premiums, it's clear that LOB 1 has grown significantly more than LOB 2.
Hence, we'll need to **account for premium growth**.

We also notice that each group of accident years is the same size which makes it easier to calculate averages across years if needed.

We'll account for the growth in premiums by dividing the ultimate claim counts by earned premium. We get the following table:

Accident Years	LOB 1	LOB 2
1991-1995	0.00001413	0.00009626
1996-2000	0.00001520	0.00009456
2001-2005	0.00001502	0.00011479
2006-2010	0.00001459	0.00011404
2011-2015	0.00001428	0.00011835
Total	0.00001464	0.00010735

We'll use the Total row as the long-term average for each line of business.
Compute the expected claim counts for each group of accident years by multiplying the total claims per $ premium for the LOB by the earned premium for the group of accident years.

Accident Year	Expected Ultimate Claim Counts	
	LOB 1	LOB 2
1991-1995	176.1	189.6
1996-2000	202.2	195.3
2001-2005	204.7	188.0
2006-2010	240.7	183.6
2011-2015	256.3	179.6
Total	1,080	936

Note that if you round the expected ultimate claim counts to the nearest integer, you'll get a slightly different chi-squared statistic to the CAS answer.

The Chi-Squared statistic is $\sum \frac{(A-E)^2}{E}$

So we have the following:

LOB 1 Chi-squared = (170 - 176.1)^2 / 176.1 + (210 - 202.2)^2 / 202.2 + (210 - 204.7)^2 / 204.7 + (240 - 240.7)^2 / 240.7 + (250 - 256.3)^2 / 256.3
= 0.8063

LOB 2 Chi-squared = (170 - 189.6)^2 / 189.6 + (172 - 195.3)^2 / 195.3 + (201 - 188)^2 / 188 + (195 - 183.6)^2 / 183.6 + (198 - 179.6)^2 / 179.6
= 8.2978

We now need the degrees of freedom. Each line of business has five sets of accident years and we estimated a single average for each.
This gives 5 -1 = 4 degrees of freedom.

From the table given for $\alpha = 0.1$ at 4 degrees of freedom the critical value is 7.779

We accept the null hypothesis for line of business 1. That is, we cannot conclude LOB 1 has shifting risk parameters.

However, for LOB 2, we reject the null hypothesis and conclude the risk parameter is changing over time.

Reading: Mahler.Credibility
Model: 2018.Q1b
Problem Type: Apply Mean-Squared-Error criterion

Given

An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

Accident Years	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	12,033,000	1,766,000	2,329,000	1,236,000	170	170
1996-2000	13,812,000	1,819,000	2,762,000	1,273,000	210	172
2001-2005	13,985,000	1,751,000	2,797,000	1,506,000	210	201
2006-2010	16,444,000	1,710,000	3,288,000	1,471,000	240	195
2011-2015	17,507,000	1,673,000	3,350,000	1,439,000	250	198
Total	73,781,000	8,719,000	14,526,000	6,925,000	1,080	936

To select an expected future claim frequency for LOB 2, the actuarial consultant has decided to assign equal weight (Z/2) to each of the most recent two groups of accident years and the remaining weight (1-Z) to the overall mean frequency.

Find b.) Calculate the expected future claim frequency per $1,000 premium for LOB 2 by first using the mean-squared-error (MSE) criterion to determine the optimal value for Z from the following three choices:

Z value	MSE
0.1	Not Provided
0.5	0.0190%
0.9	0.0164%

Solution

b.) We've added in "per $1,000 premium" into the CAS original exam question because without this, you could draw the wrong conclusion when comparing against the given MSE values. The mean-squared-error criterion means we'll select the value of Z which produces the lowest mean squared error. Consequently, we need to calculate the mean squared error when Z = 0.1.

Again, this is part of an Integrative Question, so we should be wary of additional wrinkles. In this case we'll need to account for the shift in earned premium and understand how to apply the credibility formula as well as calculate the mean squared error. All this before we can project the expected ultimate claims.
The question tells us to calculate the expected claims we need the overall average plus the two most recent groups of accident years.
This means we can calculate the expected claims for all bar the oldest and second oldest groups of accident years given as those lack data.

Further, since we're calculating the future expected claims at different points in time, we can't just use the Total row for our overall mean frequency because it may contain some years which haven't happened yet. As such, the average over "the whole period" will be a running average over all of the historical periods to date. Remember we're using Z = 0.1

Accident Years	Actual Ultimate Claims per $1,000 EP	Avg Ult Claims to date per $1,000 EP	Expected Ult Claims per $1,000 EP
1991-1995	0.0963	0.0963	NA (No prior information)
1996-2000	0.0946	0.0954	NA (Only 1 prior year of data)
2001-2005	0.1148	0.1018	0.0954
2006-2010	0.1140	0.1047	0.1021
2011-2015	0.1184	0.1074	0.1057

The CAS question wasn't very clear that the mean squared error values were also "per $1,000 of earned premium". If you converted the expected claims by multiplying by the earned premium, or if you used per $1million of premium, you get an answer that's a different order of magnitude. If this happens in an exam, it's a great clue to check your work or remember the question as potentially faulty. In the examiner's report the CAS was clear they accepted multiple answers in this situation as long as you correctly interpreted your result in the context of the given MSE values.

Now we have the expected ultimate claims per $1,000 we can apply the mean squared error formula: $mse = \sum \frac{(A-E)^2}{3}$

This formula is very close to the Chi-squared test formula. Instead of dividing by the expected value, E, we divide by the number of items we have data for to produce an average. In this case we have 3 groups of accident years.

mse = ((0.1148 - 0.0954)^2 + (0.114 - 0.1021)^2 + (0.1184 - 0.1057)^2) / 3
= 0.00022642
= 0.0226%

Comparing this result against the table given shows the lowest mse occurs at Z = 0.9

Now we've chosen the Z value, we can use all of the information available to finally answer the question and predict the future expected claim frequency. This would presumably be for accident years 2016-2020.

predicted = (0.9 / 2) * 0.1184 + (0.9 / 2) * 0.114 + (1 - 0.9) * 0.1074
= 0.1153

(Remember this is per $1,000 of earned premium).

Reading: Mahler.Credibility
Model: Source text
Problem Type: Calculate the accident year weights

Given You have the two most recent loss ratios for a line of insurance and want to combine them to calculate a rate level indication.
Assume that it is three years from the latest year of data to the average date of loss under the proposed new rates.

The following table describes the covariance structure:

Separation in Years	Loss Ratio Covariance, C(k)
0	0.00130
1	0.00060
2	0.00055
3	0.00050
4	0.00045

Find Determine the optimal least squares weights for each of the two years, assuming no external loss ratio information is used.

Solution

This is the situation described in Mahler's ratemaking example where no weight is placed on an external "grand mean".

The equation we need to use is:

$$\sum_{j=1}^{N} Z_j \cdot C(|i-j|) = C(N+\Delta-i) + \frac{\lambda}{2}$$

Here, λ, is the Lagrange multiplier.

From the question, we know N = 2 and Δ = 3.

Writing the equations out in full:

$$Z_1 \cdot C(0) + Z_2 \cdot C(1) = C(4) + \frac{\lambda}{2}$$

$$Z_1 \cdot C(1) + Z_2 \cdot C(0) = C(3) + \frac{\lambda}{2}$$

We also recall $Z_1 + Z_2 = 1$

Substituting $Z_2 = 1 - Z_1$ and adding the two equations allows us to solve for λ.

λ = 0.00095

Substituting λ into the first equation along with $Z_2 = 1 - Z_1$ yields

$Z_1 =$ 46.4%

which then gives

$Z_2 =$ 53.6%

Reading: NCCI.Circular
Model: Source Text
Problem Type: Calculate the basic premium factor

Given

Retrospective Rating Plan Parameters

(a)	Estimated Standard Premium	$500,000	
(b)	Max. Retrospective Premium Factor	130%	
(c)	Min. Retrospective Premium Factor	60%	
(d)	Loss Conversion Factor	1.12	<= c
(e)	Tax Multiplier	1.07	<= T
(f)	Loss Limit	$50,000	
(g)	Expense Ratio	0.201	
(h)	Expected Unlimited Loss Ratio	61.3%	

Find

Using the NCCI Circular CIF-2018-28 calculate the basic premium factor.

You may use the information provided below.

Policy Excess Ratio	0.582
Expected Number of Claims	20.95

Extract from the Table of Expected Claim Count Groups in Appendix A

Expected Claim Count Group	Expected Number of Claims
50	15.7 – 17.3
49	17.4 – 19.1
48	19.2 – 21.1
47	21.2 – 23.4

Extract from the Table of Policy Excess Ratio Ranges in Appendix A

Sub-table	Excess Ratio Range
14	0.485 – 0.550
15	0.551 – 0.648
16	0.649 – 0.765

Extract from Table of Aggregate Loss Factors: Sub-Table 15
Aggregate Excess Loss Factors by Expected Claim Count Group

	Expected Claim Count Group		
Entry Ratio	49	48	47
0.04	0.9622	0.9619	0.9616
0.05	0.9530	0.9527	0.9524
0.06	0.9440	0.9437	0.9434
…	…	…	…
2.32	0.0735	0.0732	0.0729
2.33	0.0726	0.0723	0.072
2.34	0.0717	0.0714	0.0711

Solution

Alice: "This is a long calculation that consists of 21 steps which are illustrated below. Work through this example carefully, referring to the wiki article when needed for explanations of each line item."

Item	Value	Description	Calculation/Notes
(1.)	$500,000	Estimated Standard Premium	
(2.)	$306,500	Expected (Unlimited) Losses	(2) = (3) * (1)
(3.)	61.3%	Expected (Unlimited) Loss Ratio	
(4.)	0.582	Policy Excess Ratio	See sub-calculation below. Yields sub-table 15.
(5.)	0.357	Excess Loss Factor	(5) = (3) * (4)
(6.)	25.6%	Expected Limited Loss Ratio	(6) = (3) - (5)
(7.)	20.95	Expected Number of Claims	See sub-calculation below. Yields count group 48.
(8.)	$100,500	Expense, Profit & Contingency excluding Taxes	(8) = (1) * (g)
(9.)	0.814	Expected Loss Plus Expense Ratio	(9) = [(2) + (8)] / (1)
(10.)	0.687	Loss & Expense in Converted Losses	(10) = (3) * (d)
(11.)	0.127	Expense, Profit & Contingency in Basic Premium	(11) = (9) - (10)
(12.)	0.561	Minimum Retrospective Premium excl. Taxes	(12) = (c) / (e)
(13.)	1.215	Maximum Retrospective Premium excl. Taxes	(13) = (b) / (e)
(14.)	0.8824	Table of Aggregate Loss Factors Value Difference*	(14) = [(9) - (12)] / [(d) * (6)]
(15.)	2.28	Table of Aggregate Loss Factors Entry Difference**	(15) = [(13) - (12)] / [(d) * (6)]
(16.)	0.05	Ratio of Losses for Minimum Retrospective Premium to Expected Limited Losses	See line-by-line wiki discussion for this figure.
(17.)	2.33	Ratio of Losses for Maximum Retrospective Premium to Expected Limited Losses	See line-by-line wiki discussion for this figure.
(18.)	0.0723	Table of Aggregate Loss Factors – Aggregate **<u>Excess</u>** Loss Factor for (17.)	AELF for (17), found in Appendix B.
(19.)	0.0027	Table of Aggregate Loss Factors – Aggregate **<u>Minimum</u>** Loss Factor for (16.)	
(20.)	0.020	Net Aggregate Loss Factor	(20) = [(18) - (19)] * (d) * (6)
(21.)	0.147	Basic Premium Factor	(21) = (20) + (11)

* Calculated to 4 decimal places to match the precision found in the Appendix B tables.

** Calculated to 2 decimal places to match the entry ratio precision found in the Appendix B tables.

<u>Policy Excess Ratio Calculation</u>

Although we gave you the Policy Excess Ratio in this question, it's conceivable you may be asked to calculate it from first principles.
It should be calculated at the State/Hazard Group level using the table approach below.

State	Hazard Group	Modified Expected Loss	Excess Ratio at Loss Limit	Expected Excess Loss	Policy Excess Ratio
X	C	106,500	0.5	53,250	
X	G	150,000	0.7	105,000	
Y	A	50,000	0.4	20,000	
Total		306,500		178,250	0.582

- The expected excess loss is the product of the modified expected loss and the excess ratio at loss limit.
- The policy excess ratio is the total expected excess loss divided by the total modified expected loss.

- The modified expected loss is the manual premium multiplied by both the experience modification (assuming the risk is also experience rated) and the expected loss ratio.

<u>Expected Number of Claims Calculation</u>

State	Hazard Group	Manual Premium	Experience Modification	Expected Loss Ratio	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
X	C	217,170			106,500	12,000	8.88
X	G	305,873			150,000	23,000	6.52
Y	A	101,958			50,000	9,000	5.56
Total			0.8	61.3%			20.95

Alice: "Remember the NCCI experience mod and expected loss ratio are the same for all states and hazard groups within a risk."

NCCI_BasicPremFactorPractice (Problem 1)

Reading: NCCI.Circular
Model: Source Text
Problem Type: Calculate the basic premium factor

Given

Retrospective Rating Plan Parameters

(a)	Estimated Standard Premium	$750,000	
(b)	Max. Retrospective Premium Factor	125%	
(c)	Min. Retrospective Premium Factor	25%	
(d)	Loss Conversion Factor	1.23	<= c
(e)	Tax Multiplier	1.14	<= T
(f)	Loss Limit	$100,000	
(g)	Expense Ratio	0.189	
(h)	Expected Unlimited Losses	$153,750	

Find

Using the NCCI Circular CIF-2018-28 calculate the basic premium factor.
You may use the information provided below.

The risk is also experience rated with experience modification factor = 0.75

State	**Hazard Group**	**Modified Expected Loss**	**Excess Ratio at Loss Limit**	**Manual Premium**	**Average Cost per Case**
X	C	32,074	0.09	208,613	10,000
X	G	106,179	0.11	690,596	21,000
Y	A	15,496	0.38	100,790	2,000

Extract from the Table of Expected Claim Count Groups in Appendix A

Expected Claim Count Group	Expected Number of Claims
51	14.3 – 15.6
50	15.7 – 17.3
49	17.4 – 19.1
48	19.2 – 21.1

Extract from the Table of Policy Excess Ratio Ranges in Appendix A

Sub-table	Excess Ratio Range
5	0.078 – 0.110
6	0.111 – 0.145
7	0.146 – 0.181

Extract from Table of Aggregate Loss Factors: Sub-Table 6
Aggregate Excess Loss Factors by Expected Claim Count Group

	Expected Claim Count Group		
Entry Ratio	51	50	49
0.16	0.8719	0.8699	0.8678
0.17	0.8649	0.8627	0.8605
0.18	0.8580	0.8557	0.8534
…	…	…	…
4.17	0.0772	0.0654	0.0545
4.18	0.0768	0.0649	0.0541
4.19	0.0763	0.0644	0.0537

Solution

Alice: "This is a long calculation that consists of 21 steps which are illustrated below. Work through this example carefully, referring to the wiki article when needed for explanations of each line item."

Item	Value	Description	Calculation/Notes
(1.)	$750,000	Estimated Standard Premium	
(2.)	$153,750	Expected (Unlimited) Losses	
(3.)	20.5%	Expected (Unlimited) Loss Ratio	(3) = (2) / (1)
(4.)	0.133	Policy Excess Ratio	See sub-calculation below. Yields sub-table 6.
(5.)	0.027	Excess Loss Factor	(5) = (3) * (4)
(6.)	17.8%	Expected Limited Loss Ratio	(6) = (3) - (5)
(7.)	16.01	Expected Number of Claims	See sub-calculation below. Yields count group 50.
(8.)	$141,750	Expense, Profit & Contingency excluding Taxes	(8) = (1) * (g)
(9.)	0.394	Expected Loss Plus Expense Ratio	(9) = [(2) + (8)] / (1)
(10.)	0.252	Loss & Expense in Converted Losses	(10) = (3) * (d)
(11.)	0.142	Expense, Profit & Contingency in Basic Premium	(11) = (9) - (10)
(12.)	0.219	Minimum Retrospective Premium excl. Taxes	(12) = (c) / (e)
(13.)	1.096	Maximum Retrospective Premium excl. Taxes	(13) = (b) / (e)
(14.)	0.7993	Table of Aggregate Loss Factors Value Difference*	(14) = [(9) - (12)] / [(d) * (6)]
(15.)	4.01	Table of Aggregate Loss Factors Entry Difference**	(15) = [(13) - (12)] / [(d) * (6)]
(16.)	0.17	Ratio of Losses for Minimum Retrospective Premium to Expected Limited Losses	See line-by-line wiki discussion for this figure.
(17.)	4.18	Ratio of Losses for Maximum Retrospective Premium to Expected Limited Losses	See line-by-line wiki discussion for this figure.
(18.)	0.0649	Table of Aggregate Loss Factors – Aggregate **Excess** Loss Factor for (17.)	AELF for (17), found in Appendix B.
(19.)	0.0327	Table of Aggregate Loss Factors – Aggregate **Minimum** Loss Factor for (16.)	
(20.)	0.007	Net Aggregate Loss Factor	(20) = [(18) - (19)] * (d) * (6)
(21.)	0.149	Basic Premium Factor	(21) = (20) + (11)

* Calculated to 4 decimal places to match the precision found in the Appendix B tables.
** Calculated to 2 decimal places to match the entry ratio precision found in the Appendix B tables.

Policy Excess Ratio Calculation

This is calculated at the State/Hazard Group level using the table approach below.

State	Hazard Group	Modified Expected Loss	Excess Ratio at Loss Limit	Expected Excess Loss	Policy Excess Ratio
X	C	32,074	0.09	2,887	
X	G	106,179	0.11	11,680	
Y	A	15,496	0.38	5,889	
Total		153,750		20,455	0.133

- The expected excess loss is the product of the modified expected loss and the excess ratio at loss limit.
- The policy excess ratio is the total expected excess loss divided by the total modified expected loss.

- The modified expected loss is the manual premium multiplied by both the experience modification (assuming the risk is also experience rated) and the expected loss ratio.

Expected Number of Claims Calculation

State	Hazard Group	Manual Premium	Experience Modification	Expected Loss Ratio	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
X	C	208,613			32,074	10,000	3.21
X	G	690,596			106,179	21,000	5.06
Y	A	100,790			15,496	2,000	7.75
Total			0.75	20.5%			16.01

Alice: "Remember the NCCI experience mod and expected loss ratio are the same for all states and hazard groups within a risk."

Reading: NCCI.InformationalExhibits
Model: Source Text
Problem Type: Produce an aggregate loss distribution from the claim count and severity distributions

Given

Count Distribution

# Claims	Probability
0	25%
1	50%
2	25%

Severity Distribution

Loss Amount	Probability
$1,000	90%
$10,000	10%

Find Produce the aggregate loss distribution from the claim count and severity distributions

Solution

We need to combine the two distributions together by considering all possible aggregate loss sizes in increasing order.

Aggregate Loss Distribution

Total Losses	Probability	Calculation	Notes
$0	25.00%	No claims	
$1,000	45.00%	1x $1,000 claim	
$2,000	20.25%	2x $1,000 claims	**<= Only 1 way to assign the claims**
$10,000	5.00%	1x $10,000 claim	
$11,000	4.50%	1x $10,000 & 1x $1,000 claims	**<= Pr(2 claims) * [Pr($10,000) * Pr($1,000) * (# ways to assign those claims)]**
$20,000	0.25%	2x $10,000 claims	**<= Only 1 way to assign the claims**

Alice: "Here are some important thoughts for when you're doing something like this under exam pressure. "

- *Remember the severity distribution applies only if a claim has occurred.*
- *Don't forgot to count the number of ways you can assign the different claim severities to the claims.*
- *It's a great idea to check your probabilities add up to 100%. This is an easy way to catch counting errors.*

NCCI_InfoSevPDF (Problem 1)

Reading: NCCI.InformationalExhibits
Model: Source Text
Problem Type: Discretize a severity distribution

Given Per-claim accident severity is modeled using a Uniform distribution on the interval [0, 10].

Find Discretize this severity distribution using evaluation points 0, 1, …, 10.

Solution

For this severity distribution the excess ratio at loss point x_i is given by $\boxed{XS(x_i) = \left(1 - \frac{x_i}{10}\right)^2}$ *Alice: "You should check you can derive this – it's a great application of Bahnemann and a primer for IQs."*

Now form a table with a row for each of the evaluation points

Evaluation Point, x_i (1)	$XS(x_i)$ (2)	LEV_i (3)	LIL_i (4)	CDF (5)	PDF (6)
0	1.00	0.00	0.00	0.05	0.05
1	0.81	0.95	0.95	0.15	0.10
2	0.64	1.80	0.85	0.25	0.10
3	0.49	2.55	0.75	0.35	0.10
4	0.36	3.20	0.65	0.45	0.10
5	0.25	3.75	0.55	0.55	0.10
6	0.16	4.20	0.45	0.65	0.10
7	0.09	4.55	0.35	0.75	0.10
8	0.04	4.80	0.25	0.85	0.10
9	0.01	4.95	0.15	0.95	0.10
10	0.00	5.00	0.05	1.00	0.05

Notes

(2) = (1 - (1) / 10)^2

(3) = [1 - (2)] * (Average Unlimited Severity) *Alice: "The Average Unlimited Severity is just* E[X]."

(4) = $LEV_i - LEV_{i-1}$

(5) = 1 - $LIL_{i+1} / (x_{i+1} - x_i)$

Alice: "Notice $LEV_i \leq x_i$ *and* $LEV_i - LEV_{i-1} \geq LEV_{i+1} - LEV_i$*. i.e.* LIL_i *is a decreasing function of* x_i*"*