12. (3.5 points)

Undeveloped losses follow a uniform distribution between $0 and $500. Each loss has an equal likelihood of developing such that it is multiplied by either 0.75 or 1.25.

a. (1 point)

Calculate the excess ratio at $400 for undeveloped losses.

b. (1.5 points)

Calculate the excess ratio at $300 for developed losses.

c. (1 point)

Suppose instead that loss multipliers are uniformly distributed between 0.75 and 1.25.

Determine whether the excess ratio at $300 for developed losses will be higher than, equal to, or lower than the excess ratio calculated in part b above. Do <u>not</u> attempt to calculate the new excess ratio.

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Question 12:

Model Solution 1

a.

$$R(400) = \frac{\int_{400}^{500} (x-400)(\frac{1}{500})dx}{E[X]} \qquad E[X] = \frac{0+500}{2} = 250$$

$$= \frac{10}{250} = \underline{\underline{0.04}} \qquad \int_{400}^{500} (x-400)(\frac{1}{500})dx = \frac{1}{500}(\frac{x^2}{2} - 400x)\Big|_{400}^{500} = 10$$

b.

$$\hat{R}(300) = (1/2)(0.75)R(300/.75) + (1/2)(1.25)R(300/1.25)$$

$$= (1/2)(.75R(400) + 1.25R(240)) \qquad \int_{240}^{500} (x-240)(\frac{1}{500})dx = \frac{1}{500}(\frac{x^2}{2} - 240x\Big|_{240}^{500}$$

$$= (1/2)(.75*.04 + 1.25*.2704) \qquad = 67.6$$

$$= \underline{\underline{0.184}} \qquad R(240) = \frac{67.6}{250} = 0.2704$$

c.

Old multipliers: $E[X] = .5(.75) + .5(1.25) = 1$
$E[X^2] = .5(.75^2) + .5(1.25^2) = 1.0625$
$Var(X) = 1.0625 - 1^2 = .0625$

$\mu = 1 \quad \sigma = \sqrt{.0625}$ Coefficient of Variation = $\frac{\sqrt{.0625}}{1} = 0.25$

New multipliers

$$E[X] = \frac{.75 + 1.25}{2} = 1$$

$$E[X^2] = \int_{.75}^{1.25} x^2(\frac{1}{1.25 - .75}) = \frac{1}{0.5}(\frac{x^3}{3}\Big|_{.75}^{1.25} = 1.0208$$

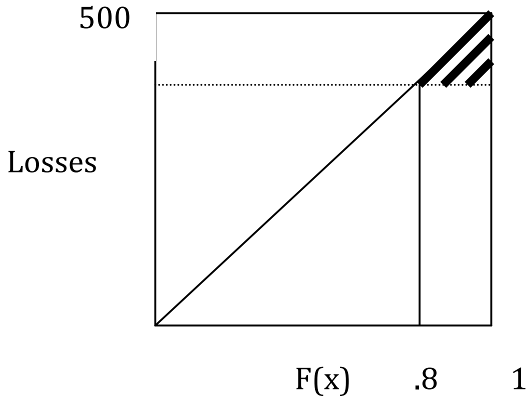
$Var(X) = 1.0208 - 1^2 = 0.0208$

$\mu = 1 \quad \sigma = \sqrt{.0208}$ Coeff of Variation = $\frac{\sqrt{.0208}}{1} = 0.144$

Since the coefficient of variation for the loss multipliers from 0.75 to 1.25 based on the uniform distribution is smaller, the impact of dispersion will be less & thus the excess ratio will be lower.

Model Solution 2 (parts a and b only)

a.



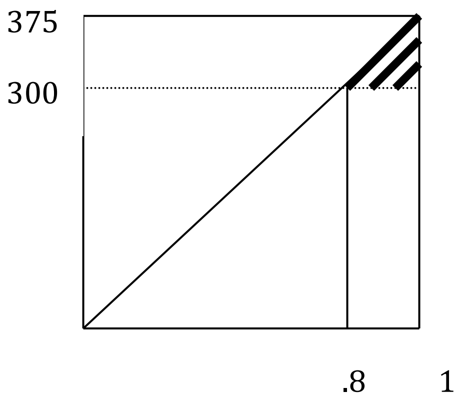
R(400) = shaded area / total area

$$= \frac{(\frac{1}{2})(.2)(100)}{(\frac{1}{2})(1)(500)}$$

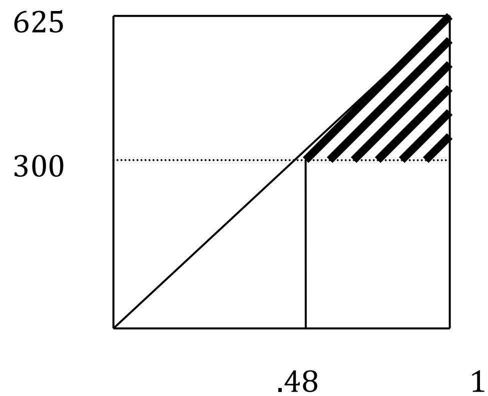
R(400) = .04

b.

Developed, .75



Developed, 1.25



$$R(300) = \frac{(\frac{1}{2})(.2)(75)}{(\frac{1}{2})(1)(375)} = .04$$

$$R(300) = \frac{(\frac{1}{2})(.52)(325)}{(\frac{1}{2})(1)(625)} = .2704$$

$$\hat{R} = \frac{(\frac{1}{2})(187.5)(.04) + (\frac{1}{2})(312.5)(.2704)}{(\frac{1}{2})(187.5) + (\frac{1}{2})(312.5)} = \boxed{.184}$$

Model Solution 3 (part b only)

After development, we have two uniform distributions with 50%, 50% chance. One is between 0 & 375 i); the other is 0 & 625 ii)

uniform i) $\int_{300}^{375} G(x)dx = \int_{300}^{375} \frac{375-x}{375} dx = 75 - \left.\frac{x^2}{750}\right|_{300}^{375} = 7.5$

uniform ii) $\int_{300}^{625} G(x)dx = \int_{300}^{625} \frac{625-x}{625} dx = 325 - \left.\frac{x^2}{1250}\right|_{300}^{625} = 84.5$

uniform i) Average loss = $\frac{0+375}{2}$ = 187.5

uniform ii) Average loss = $\frac{0+625}{2}$ = 312.5

Excess ratio R(300) = $\frac{7.5*0.5+84.5*0.5}{187.5*0.5+312.5*0.5}$ = 18.4%

Model Solution 4 (part c only)

It will be lower, essentially because the uniform distribution will be less dispersed vs the distribution above. (The dist above can only be either endpoints => greater deviation from the mean vs a uniform that gives weight to intermediate values). Decreasing the CV (= $\frac{\sigma}{\mu}$ = measure of dispersion) will decrease the XS ratio at higher limits like 300.

Examiner's Comments:

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Part a was very straightforward and most candidates did very well.

On part b, most candidates did well and utilized the developed loss excess ratio formula correctly. The most common mistakes were:

- Confusing multipliers with divisors;
- Adding an extra term in the formula to account for no development (which was irrelevant to the problem); and
- Simply averaging R(400) and R(240) instead of weighting them by the multipliers

On part c, in order to get full credit, the candidate needed to calculate the coefficient of variation (CV) of each distribution or explain in detail why the CV of the uniform distribution would be less than that of the discrete distribution (and therefore that the excess ratio would be less). Most candidates did not mention the relationship that the excess ratio has with the CV of the loss multipliers and simply stated that the dispersion or variance would be lower for the uniform distribution without offering any explanation or support as to why that would be the case.

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