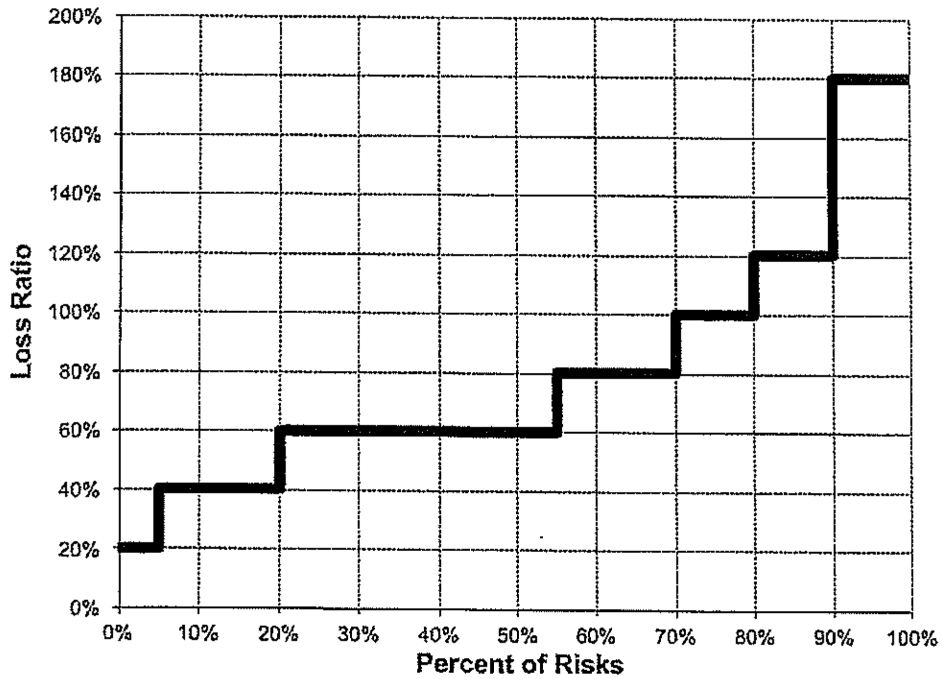
21. (2 points)

A group of similar risks has an average loss ratio of 80%. The following Lee diagram depicts this group of risks:



a. (1.5 points)

Calculate the insurance savings at an entry ratio of 1.125.

b. (0.5 point)

The risks above the 90th percentile have their losses restated, significantly increasing the loss ratio. Describe the change to the insurance savings at an entry ratio of 1.125.

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<u>Question 21:</u>

Model Solution 1

A) ELR = 0.8, Entry ratio = 1.125
LR = 1.125*(0.80) = 0.90

Insurance Savings @ entry ratio 1.125 (before standardizing)
=0.05 (0.9-0.2) + (0.2-0.05)(0.9-0.4) + (0.55-0.2)(0.9-0.6) + (0.7-0.55)(0.9-0.8) = 0.23

After standardizing -> 0.23/0.80 = 0.2875

B) Since the average loss ratio increases, entry ratio of 1.125 would correspond to a higher loss ratio, hence increasing the insurance savings.

Model Solution 2

A) $R = 1.125 = L/80\% \rightarrow L = 90\%$

Loss Ratio of 180% $\rightarrow R = 2.25$
Loss Ratio of 120% $\rightarrow R = 1.5$
Loss Ratio of 100% $\rightarrow R = 1.25$

$\Phi(1.5) = (10\%)(2.25 - 1.5) = 0.075$
$\Phi(1.25) = (20\%)(1.5 - 1.25) + \Phi(1.5) = 0.05 + 0.075 = 0.125$
$\Phi(1.125) = (30\%)(1.25 - 1.125) + \Phi(1.25) = 0.0375 + 0.125 = 0.1625$

$\Psi(1.125) = \Phi(1.125) + R - 1 = 0.1625 + 1.125 - 1 = 0.2875$

B) Since $\Psi = \Phi + R - 1$, then the savings will increase because an increase to the highest decile will increase the charge, all else being equal. The average loss ratio will also increase, making R = 1.125 correspond to a higher loss ratio. This increases the area between R and F(x), further increasing the savings.

Examiner's Comments:
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Part a:

There were 2 principal methods to calculate the insurance savings. The candidate had the option to either calculate the savings directly, or to first calculate the insurance charge and then use the relationship between the insurance charge and savings to produce the correct solution.

Using the former method, the candidate would need to determine the area above the curve F(x) and below the loss ratio. The loss ratio is calculated as the product of the expected loss ratio (80%) and the entry ratio (1.125), or 90%. The area is equal to 0.23. The candidate must then standardize the savings by dividing the area by the expected loss ratio of 80%, producing the insurance savings of 0.2875 or 28.75%.

Using the latter method, the candidate would need to first calculate the insurance charge, which is the area below the curve F(x) and above the loss ratio of 90% (as described above). This area is equal to 0.13. To calculate the insurance charge, the candidate must standardize by dividing the area by 0.80, producing the insurance charge of 0.1625 or 16.25%. The savings is derived by using the formula:

$\Psi = \Phi + r - 1 = 0.1625+1.125-1=0.2875$ or 28.75%.

Common errors made by candidates included:

- Failing to standardize the savings (or the charge) by the 80% loss ratio;
- Providing only the insurance charge, rather than the savings;
- Not calculating the area between the curve and the loss ratio correctly.

Part b:
Candidates received full credit if <u>both</u> of the following were provided in the solution:

- The direction that the insurance savings would move; and
- The correct rationale for this change.

One potential solution would explain that because the 90th percentile corresponds to a 120% loss ratio, which is higher than the 1.125 entry ratio, there will be an increase to the insurance charge. Referencing the formula $\Psi = \Phi + r - 1$, the candidate can demonstrate that an increase to the charge would imply a simultaneous increase in the savings, all else being equal.

Alternatively, the candidate could have described that an increase in the average loss ratio would imply a higher loss ratio at the entry ratio of 1.125. This would in turn increase the area below the curve F(x) and above the loss ratio, implying an increase in savings.
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