18. (4 points)

An actuary is pricing a Large Dollar Deductible (LDD) workers compensation policy. To price the excess loss portion, an actuary uses a blend of empirical data and a fitted curve to estimate excess loss pure premium factors. The cut-off for empirical data is $250,000. Using the data below, calculate an LDD premium for a policy with a $500,000 deductible and no aggregate limit.

Historical adjusted loss and ALAE for similarly sized risks:

Loss and ALAE	Observed Percentage Of Claim Counts
8,000	80%
100,000	11%
250,000	6%
500,000	2%
1,000,000	0.7%
2,000,000	0.3%

Total expected losses per risk = 55,400

Excess ratios based on fitted mixed Exponential-Pareto distribution on losses Truncated and shifted at $250,000:

Entry Ratio	Excess Ratio
0.1	0.92
0.2	0.84
0.3	0.73
0.4	0.69
0.5	0.65
0.6	0.60
0.7	0.55
0.8	0.51
0.9	0.47
1.0	0.44

Standard premium	$100,000
Loss based assessment (% of loss and ALAE)	4%
ULAE (% of loss and ALAE)	8%
General expenses (% of standard premium)	5%
Credit risk (% of standard premium)	5%
Acquisition expense (% of net premium)	8%
Tax (% of net premium)	3%
Profit Load (% of net premium)	5%

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Question 18:

Model Solution 1

$$\hat{R}(500) = R(250)xR_Y(500-250)$$

$$R(250) = 1 - \frac{8(.80)+100(.11)+250(.09)}{55.4} = 0.2798$$

$$R_Y(500-250): r = \left(\frac{500-250}{AvgTruncated\ \&\ Shifted\ @250K}\right)$$

$$= \frac{250}{\frac{(500-250)x.02+(1000-250)x.007+(2000-250)x.003}{.02+.007+.003}} = 0.48$$

$R_Y(0.48) = 0.658$ (interpolate from table)

$\hat{R}(500) = 0.2798x0.658 = 0.1841$

$Ee = 0.1841x55{,}400 = 10{,}199$

$$LDD\,\text{Prem} = \frac{Ee+E(LBA+ULAE)+SP(GO+CR)}{1-A-T-p} = \frac{10{,}199+55{,}400(4\%+8\%)+100{,}000(5\%+5\%)}{1-8\%-3\%-5\%}$$

LDDPrem = 31,961

Model Solution 2

R(250) = Losses above 250,00 / Total Losses

Losses above 250K =

2%*(500K – 250K) + 0.7%*(1000K – 250K) + 0.3%*(2000K – 250K) = 15.5K

Total Losses = 55.4K

R(250) = 15.5K / 55.4K = 0.28

Above truncation 250K, mean = 15.5K / (.02+.007+.003) = 516.67K

➔r = 250K / 516.67K = 0.4839

Interpolation: $\left(\frac{0.4839-0.4}{0.1}\right)x0.65 + \left(1-\frac{0.4839-0.4}{0.1}\right)x0.69 = 0.656$

➔XL= 0.656 x 0.28 = 0.184

LDDPrem=

$$\frac{ELx(ULAE+LBA+XL)+SPx(GO+CR)}{1-A-T-p} = \frac{55{,}400x(.08+.04+0.184)+100{,}000x(.05+.05)}{1-.08-.03-.05}$$

=31,941.52

Model Solution 3

LDDPrem= $\frac{ELx(ULAE + LBA + XL) + SPx(GO + CR)}{1 - A - T - p}$

E(loss) = 55,400

$$R(250{,}000) = 1 - \frac{8000(.80) + 100000(.11) + 250000(.06 + .02 + .007 + .003)}{55{,}400} = 0.2798$$

e(250,000)=

$$\frac{0.02(500{,}000 - 250{,}000) + 0.007(1{,}000{,}000 - 250{,}000) + 0.003(2{,}000{,}000 - 250{,}000)}{0.03} = \frac{15{,}500}{0.03} = 516{,}667$$

$r = \frac{500{,}000 - 250{,}000}{516{,}667} = 0.4839 \rightarrow$ round to 0.5 and use xs ratio at 0.

$\therefore R(500{,}000) = R(250{,}000)xR(500{,}000 - T) = 0.2798x0.65 = 0.18187$

$$\therefore LDD\,\mathrm{Pr}\,em = \frac{55{,}400(0.18187 + 0.08 + 0.04) + 100{,}000(0.05 + 0.05)}{1 - 0.08 - 0.03 - 0.05} = 31{,}814$$

Examiner's Comments:

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Generally the candidates did well on this question. Most understood that they needed to calculate an empirical piece, and a model piece, and combine them. Probably the hardest part is getting the formulas right, and if they did, the calculation was pretty straightforward. We did see some papers that showed answers with no discernible work.

There were 5 main calculations:

- R(250) – the empirical excess ratio at $250,000
- Average Truncated and Shifted losses at $250,000
- Entry Ratio – to use in the table of excess ratios for the $500,000 level
- R(500) = XL – the total excess loss ratio
- LDD Prem - being able to remember the formula correctly, and substituting in the figures given for XL, expenses & profit, and calculating the final answer.

Most candidates received full credit for the R(250) calculation. Common errors here were for calculating the losses limited to 250, rather than the excess.

Average Truncated and Shifted (AT&S) was the most problematic of the calculations. Some skipped it completely, not realizing that this was the denominator in the entry ratio calculation. Others calculated the limited instead of the excess. There was actually a short cut here, as the numerator of the R(250) and the AT&S are the same.

The Entry Ratio calculation was straighforward if the AT&S was calculated correctly. If the AT&S was not correctly calculated, most candidates gave the correct numerator of the entry ratio. Then, being able to select the appropriate excess ratio from the table was straightforward. Rounding the entry ratio to 0.50 to select a value in the table was given full credit.

Nearly all candidates knew that the final XL was the product of two figures, and most knew that it was the product of R(250) and the figure they looked up in the entry ratio table.

The LDD Premium formula was a challenge to quite a few candidates; nearly all got the expense portions correct, but were stymied by the portion that involved the XL. While most gave the correct formula for an LDD premium, several seemed to confuse it with an Excess premium.

Depending how and where the candidate rounded their calculations, the final answer could vary from roughly 31,700 to roughly 32,100. If work was shown and the answer fell in this range, the candidate received full credit.
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