18. (2.5 points)

An actuary prices a retrospectively rated policy with the following provisions resulting in a balanced plan:

Minimum premium:	17,500
Maximum premium:	80,000
Expense provision:	8,125
Loss conversion factor:	1.25

- Aggregate losses follow a uniform distribution between $0 and $100,000
- No taxes

After pricing the policy, the actuary discovers an error in the original loss distribution and determines that losses should instead follow a uniform distribution between $0 and $90,000. The actuary decides to re-balance the plan based on the corrected distribution while still maintaining the same minimum and maximum premium.

Calculate the loss at minimum premium and loss at maximum premium that re-balance the plan.

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QUESTION 18

TOTAL POINT VALUE: 2.5 **LEARNING OBJECTIVE: B5**

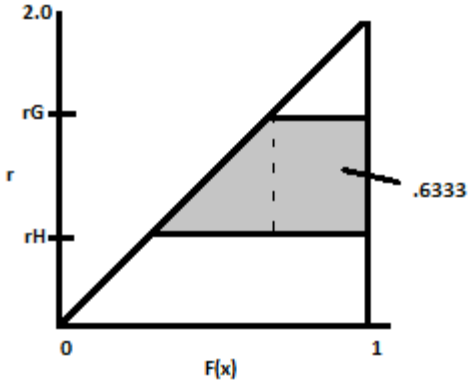
SAMPLE ANSWERS

Sample 1

$$E = 45k$$

$$r_G - r_H = \frac{G-H}{cE} = 1.111$$

$$X_H - X_G = \frac{e+E-H}{cE} = .6333$$



$$(r_G - r_H)(1 - {}^{r_G}\!/_{2}) + \frac{(r_G - r_H)\frac{(r_G - r_H)}{2}}{2} = .6333$$

$$r_g = 1.111 + r_H$$

$$1.111\left(1 - \frac{1.111 + r_H}{2}\right) + \frac{1.111(1.111/2)}{2} = .6333$$

$$r_H = .3044$$

$$r_G = 1.4155$$

$$L_H = r_H * E = .3044 * 45k = 13.7k$$

$$L_G = r_G * E = 1.4155 * 45k = 63.7k$$

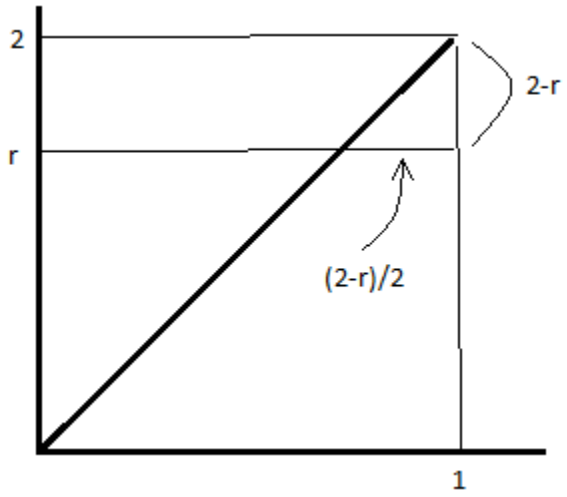
Sample 2

$$E = \frac{0 + 90,000}{2} = 45,000$$

$$\phi(r_H) - \phi(r_G) = \frac{e + E - H}{cE} = \frac{8,125 + 45,000 - 17,500}{1.25 \times 45,000} = 0.6333$$

$$r_G - r_H = \frac{G - H}{cE} = \frac{80,000 - 17,500}{1.25 \times 45,000} = 1.1111$$

$$r_G = 1.1111 + r_H$$



$$\phi(r) = \frac{1}{2}(2 - r)\frac{(2 - r)}{2} = \frac{(2 - r)^2}{4}$$

$$\phi(r_H) - \phi(r_G) = \frac{(2 - r_H)^2}{4} - \frac{(2 - r_G)^2}{4} = 0.6333$$

$$(2 - r_H)^2 - (2 - (1.1111 + r_H))^2 = 0.6333 \times 4$$

$$2^2 - 4r_H + r_H^2 - 0.8889^2 + 2 \times 0.8889 r_H - r_H^2 = 0.6333 \times 4$$

$$\frac{4 - 0.8889^2 - 0.6333 \times 4}{4 - 2 \times 0.8889} = r_H = 0.3045$$

$$L_H = r_H \times E = 0.3045 \times 45,000 = 13,702.5$$

$$r_G = 0.3045 + 1.1111 = 1.4156$$

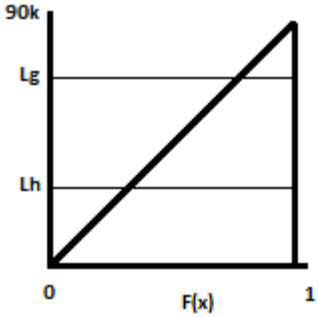
$$L_G = 63,702$$

Sample 3

$$E = 45k$$

$$L_G - L_H = \frac{G-H}{cT} = \frac{80-17.5}{1.25*1} = 50k$$

$$(X_H - X_G)E = \frac{(e+E)-H/T}{cE} * E = \frac{(8.125+45)-17.5}{1.25*45} * E = .6333 * E = 28.5$$



$$X_G E = \frac{1}{2}(90-L_G)\frac{90-L_G}{90} = \frac{1}{180}(90-L_G)^2$$

$$X_H E = \frac{1}{180}(90-L_H)^2$$

$$\frac{1}{180}(90-L_H)^2 - \frac{1}{180}(90-L_G)^2 = 28.5$$

$$L_G - L_H = 50 \ (from\ above)$$

$$(90-L_H)^2 - (90-50-L_H)^2 = 5130$$

$$L_H = 13.7k$$

$$L_G = 63.7k$$

EXAMINER'S REPORT

This was a challenging question, and candidates generally performed poorly. The most difficult aspect was relating the "left-hand side" of the 2nd balance equation to the aggregate loss distribution, that is, expressing $\phi(r_H) - \phi(r_G)$ in terms of r_G and r_H (or L_G and L_H). Very few candidates made this connection; in fact, the majority of candidates failed to realize that the balance equations were necessary in order to re-balance the plan.

Many candidates calculated the guaranteed cost premium, which was not needed to solve the problem. Many candidates also attempted to find the basic premium, which was not needed to solve the problem.

Other common mistakes included:

- Confusing G and H (80k and 17.5k) with L_G and L_H
- Confusing the expense provision (8.125k) with the basic premium
- Using the original loss distribution of U[0, 100k] rather than the revised distribution of U[0, 90k]
- Coming up with the system of equations (that is, two equations with two unknowns) but failing to correctly solve the system (candidates were only marginally penalized for not completing this step)