4. (2.25 points)

An actuary is reviewing an account that has been with the company for over ten years.

Given the following:

- The claim frequency for this account follows a Poisson distribution, with $\lambda = 0.012$
- The recorded frequency for the last five years is as follows:

Year	Exposures	Frequency
2010	9,500	0.011
2011	11,000	0.010
2012	13,000	0.013
2013	10,500	0.012
2014	12,000	0.010

- The critical value for the relevant Chi-squared distribution is 9.49

a. (1.5 points)

Use the Chi-squared test to evaluate whether the claim frequency is shifting over time.

Include the hypotheses, test statistic, and provide an interpretation of the result.

b. (0.75 points)

Fully describe another method for determining whether claim frequency is shifting over time.

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QUESTION 4

Total Point Value: 2.25 **Learning Objective: A1**

Sample Answers

Part a: 1.5 points

Sample 1

Chi-square test statistic: $X^2 = \sum_i \frac{(Ai-Ei)^2}{Ei}$

The null hypothesis H_0, is that the claim frequency is <u>not</u> shifting over time

Actual # claims=exposures * frequency

Expected # claims = exposures * λ

Year	Actual	Expected
2010	104.5	114
2011	110	132
2012	169	156
2013	126	126
2014	120	144

Test statistic is $(104.5-114)^2 / 114 + \ldots + (120-144)^2 / 144 = 0.792 + 3.667 + 1.033 + 0 + 4 = 9.542$

Since TS > critical value of 9.49, I would reject the null hypothesis and say that claim frequency is shifting over time.

Sample 2

Ho: Frequency is not shifting over time

$$X^2 = \sum w \frac{(freq - \lambda)^2}{\lambda}$$

$X^2 = 9500(.011-.012)^2/.012 + 11000(.01-.012)^2/.012 + 13000(.013-.012)^2/.012 + 10500(.012-.012)^2/.012 + 12000(.01-.012)^2/.012 = 9.542$

9.542>9.49, reject Ho ➔Frequency is shifting

Part b: 0.75 points

Sample 1

Lagged Year Correlations:

1. Calculate the average correlation in frequency for each one year lag (2009-2010, 2010-2011, etc)
2. Do this same average correlation calculation for two year lag, three year lag, etc.
3. If the average correlation decreases as lag increases, frequency is likely shifting over time.

Sample 2

Another method would be a correlation test. For every pair of lag years, for example 1 year lag (2000-2001, 2002-2003, etc.) Calculate the correlation between frequencies of those years and average all the correlations of the pairs to get the 1 year lag correlation. Do this for all lags, like 2-year lag (2000-2002, 2001-2003, etc.), 3-year lag (2000-2003, 2001-2004, etc.), etc. If the correlation decreases as the lag increases, then can conclude that the parameters are shifting over time.

Examiners Report

Part a:

Crucial in part A was setting up the test by stating the hypothesis, noting the formula for the Chi-square statistic, calculating the items in the formula & the statistic, and reaching the correct conclusion by comparing the calculated statistic to the given critical-value by declaring whether you accept or reject the hypothesis.

The question specifically asked for candidates to state their hypotheses, calculate the test statistic, and state their interpretation of the test result. Most candidates calculated the test statistic and stated a conclusion, but many candidates forgot to state the null hypothesis. Some candidates did not show sufficient work and it was difficult to determine knowledge of the material if a wrong test statistic was calculated or a wrong conclusion was drawn.

If the candidate made a calculation error, but was able to draw the proper conclusion based on that error, they got credit for their conclusion, but not the calculation. For example, if the candidate miscalculated the test statistic to something smaller than 9.49 and accepted their null hypothesis, they got credit for the conclusion; and if they miscalculated to something larger than 9.49 and rejected their null hypothesis, they got credit for the conclusion. This assumes that the candidate stated the hypothesis correctly.

Part b:

This part was looking for the correlation test described by Mahler on page 235, 4th paragraph. Most candidates were able to do this, some in a few sentences, some a little longer. Key words sought were "correlation", "pairs of years", "average correlation for all pairs with same lag", "correlation decreases as lag increases". We did see a handful of alternate answers, but most of these did not meet the "fully describe" statement in the question.

Where candidates lost credit, they were generally unclear in describing that they were calculating the corrections between pairs, or did not mention that they should take the average of all correlations between pairs of the same lag. It was not enough to say that correlation changes, or that it decreases over time. Neither gives the impression that the candidate knew that the correlation decreases as lag increases. We saw answers using terms like "difference", "variance", "covariance", and "confidence interval" instead of "correlation". We also saw answers that implied a correlation among more than a pair of years.