8. (2.5 points)

An actuary is working with ground-up historical loss data and is considering fitting one continuous curve to this data to calculate ILFs for higher limits.

The last time such an analysis was conducted, empirical losses were used to determine ILFs directly without fitting a continuous curve to the data.

a. (1 point)

Provide two shortcomings of using empirical data to determine ILFs and briefly describe how curve fitting may overcome each of these shortcomings.

b. (1.5 points)

There is a concern that fitting one continuous curve to the entire distribution of losses will overstate losses over certain intervals and understate losses over other intervals. Propose and fully describe a solution that addresses this concern while still incorporating an element of curve fitting in the solution.

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Total Point Value: 2.5 **Learning Objective: B1, B2**

Sample Answers

Part A: 1.00 points

Sample 1

Empirical losses at higher limits may be volatile. Curve fitting can smooth out the volatility.

Empirical losses may not reach maximum policy limits, so no factor can be calculated (free cover). Curve fitting can extrapolate losses to higher limits.

Sample 2

There can be gaps in the data if the empirical data is thin at higher sizes of loss.

There can also be cluster points in the data around round numbers.

Curve fitting can smooth over cluster points and provide information where gaps occur to reduce impact from having gaps.

Sample 3

Losses used in fitting the curve may develop further. Curve fitting can take loss development (and even the dispersion in development) into consideration

The credibility at the high end of the distribution is a concern. Curve fitting fits a curve that maximizes the likelihood of all reported losses.

Part b: 1.50 points

Sample 1

- ILFs below a certain threshold can be determined directly from the data.
- It allows us to rely on the actual data for the lower layers where there is a larger volume of data subject to random fluctuations. ILFs above that threshold can be estimated using curve fitting (e.g. a simple or mixed distribution) to more accurately estimate losses at higher dollar amount intervals.
- The threshold above which curve fitting should be employed should be selected to permit the maximum reliance on reported data while still retaining enough data above the threshold to permit reasonable fitting of a loss distribution. It should be a round number prior to the 'thinning out' of the data.
- This method provides a smooth transition from relying on data for lower accident limits to relying on a fitted curve to provide some information at higher accident limits.

Sample 2

One would use empirical data where credible; cutoff at a level where there is just enough higher limit losses to fit a curve (perhaps $100k). Then we can fit an Exponential curve for the first layer of XS losses; and this will provide a smooth join to the empirical loss distribution. Finally, we can use the Pareto distribution to fit the highest layer of losses.

Sample 3

Up to a certain cutoff point ($100k), use empirical data directly with the empirical distribution. Above the cutoff point, shift and truncate the data, then fit a mixed Exponential/Pareto distribution to the shifted and truncated data. This approach allows us to use actual data as much as possible at layers where it is sufficiently credible, while preserving enough data above the truncation point to fit curves. The Exponential component of the mixture is light-tail and reflects losses just over the truncation point, while Pareto is heavy-tail and best reflects high loss layers. The shifted and truncated mixed Exponential/Pareto distribution can be smoothly joined to the empirical distribution up to the truncation point, yielding consistent results.

Examiners Report

Part a:

- In order to receive full credit a candidate needed to:
  - Identify two distinct issues with empirical ILFs that could be resolved by curve fitting
  - Provide a brief description of how curve fitting would overcome.
- Common errors made by candidates were as follows:
  - Giving two valid shortcomings but failing to describe how curve fitting would overcome the shortcomings
  - Stating only that "curve fitting solves this problem" without description of how.
- Candidates are expected to be able to draw on a list of problems with empirical ILFs that can be addressed through curve fitting.
- Note that ILFs built directly from empirical data using an approach as described in Lee would not fail the consistency test. Issues with failing consistency test were therefore not given credit.
- A few items that were noted as problems with empirical data also cause the same problem with curve fitting and can be solved in similar ways with use of empirical data. For example, selection issues related to differences in severity profile for risks purchasing different limits was often given as an issue with the solution being to make separate curves for different limits. This process is also possible using empirical ILFs by segmenting that data similarly so no credit was given for answers of this type.
- Credit was given for recognizing development as an issue with empirical data but credit was only given for how curve fitting helps if it incorporated an element of using a distribution for dispersion of development. Saying to build the curve only on mature data was not acceptable as the same solution could be applied in an empirical method.

- Issues with policy limits causing a bias in the distribution were acceptable if accompanied by an explanation that the calculation of variance for use in risk load was improved through fitting a theoretical curve.

Part b:

Candidates needed to mention selection of a truncation point below which empirical data should be used directly and above which a curve should be fit. The solution should explain why this is done. Most candidates failed to identify that the empirical distribution and curve should be joined smoothly together.

Fully illustrating a mixed Exponential/Pareto curve without addressing reliance on empirical data below the truncation point received no credit.