1. (2.75 points)

A group of insureds have different expected claim frequencies. The number of insureds claim-free for the past *t* years is as follows:

Expected Claim Frequency	**t=0**	**t=1**	**t=2**	**t=3**
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Determine whether the variation of an individual insured's chance for an accident changes over time.

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QUESTION: 1	
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE(S): A1
SAMPLE ANSWERS	

Sample 1

Given:

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Calculate claims at time t:

Claims	t=0	t=1	t=2	t=3
0.05	2,500 = .05 x 50,000	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600

Calculate average frequency at time t:

0.1000 = 12,500 / 125,000	0.0971	0.0943	0.0915

Calculate frequency relative to t=0:

1.0000	0.9712 = 0.0971 / 0.1000	0.9426	0.9149

Credibility:

0.0288 = 1 - 0.9712	0.0574	0.0851

Credibility relative to t=1:

1.9963 = 0.0574 / 0.0288	2.9591

Variation of insureds' chances of accident are stable if credibility is proportional to the number of years of experience. Since the ratios of credibility are very nearly equal to 3 and 2, we conclude that the variation of an insured's chance of accident is not changing over time.

Sample 2

	(1)	(2)	(3)	(4)	(5)
n	# Claim free n or more years	Expected Claims	Frequency	Relative Frequency	Z
3	94,000	8,600	0.0915	0.9525	0.0475
2	198,500	18,450	0.0929	0.9677	0.0323
1	311,500	29,425	0.0945	0.9835	0.0165
Total	436,500	41,925	0.0960	1	

Expected claims:

- t=3: 44,000 x 0.05 + 36,000 x 0.10 + 14,000 x 0.20 = 8,600
- t=2: 45,000 x 0.05 + 43,000 x 0.10 + 16,500 x 0.20 = 9,850
- t=1: 47,500 x 0.05 + 45,000 x 0.10 + 20,500 x 0.20 = 10,975
- Total: 186,500 x 0.05 + 174,000 x 0.10 + 76,000 x 0.20 = 41,925

(3) = (2)/(1)
(4) = (3)/$(3)_{\text{Total}}$
(5) = 1 – (4)

If the variation of an insured's chance for an accident is not changing over time, then the 3-year credibility/1-year credibility will be approximately equal to 3 and the 2-year credibility/1-year credibility will be approximately equal to 2.

3+ year Z / 1+ year Z = 0.0475 / 0.0165 = 2.88
2+ year Z / 1+ year Z = 0.0323 / 0.0165 = 1.96
The ratios are approximately 3 and 2; the chance for accident is stable.

Sample 3

Credit was given for an approach that evaluated the correlation between different lags for either the relative number of insureds in each class or the frequency at each time period.

Correlation between relative number of insured in each class at different lags:

- Calculate relative distribution of insured by class (note that total insureds by class could be used for the approach below and will result in the same correlation values and conclusions):

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	40%=50K/125K	42%	43%	47%
0.10	40%	40%	41%	38%
0.20	20%	18%	16%	15%
Total	100%	100%	100%	100%

- Calculate correlations between each lag vector, and calculate averages by lag 1, 2, and 3:

lag = 1	t=0 & t=1 0.9965	t=1 & t=2 0.9998	t=2 & t=3 0.9806	Average 0.9923
lag = 2	t=0 & t=2 0.9980	t=1 & t=3 0.9845		Average 0.9912
lag = 3	t=0 & t=3 0.9663			Average 0.9663

- Determine whether the correlation is decreasing as the lag length is increasing. In the above example, this is true. Therefore we conclude that the parameter is changing over time.

Sample 4

Credit was also given to students that used the correlation approach but calculated expected claim counts, or actual frequencies, and then calculated whether these correlations were changing over time. The correlations for both are shown below. In both cases the student will also conclude that the correlation is changing as the time lag increases, and that therefore the risk parameters are changing.

Claim count calculation:

Average correlation test using calculator tables:

For lag 1 = r(0,1) = 0.9842; r(1,2)=0.9456; r(2,3)=0.9954; average=0.9750

For lag 2 = r(0,2) = 0.8730; r(1,3) = 0.9909; average = 0.8914

For lag 3 = r(0,3) = 0.8220; average = 0.8220

Downward trending average r correlation as lag increases.

Conclusion: Yes, variation of insured's chance of an accident is changing

Actual frequency calculation:

Actual Claim Free Frequency	**t=0 to t=1**	**t=1 to t=2**	**t=2 to t=3**
0.05	0.9500 =4,750/5,000	0.9474	0.9778
0.10	0.9000	0.9556	0.8372
0.20	0.8200	0.8049	0.8485

Calculate correlations between lags:
For lag 1, corr(t1, t2) = 0.903; corr(t2,t3) = 0.39; average of **0.646**
For lag 1, corr(t1, t3) = **0.748**

Sample 5
Partial credit was also given to students that stated that the Chi Squared test may be used.

Do a Chi Squared test with Chi Squared = $\sum(Actual - Expected)^2/Expected$
Across 12 cells with 11 degrees of freedom. If we reject, that means the parameters are changing over time.

EXAMINER'S REPORT

Candidates were expected to use credibility concepts to evaluate underlying risk parameters that may be changing over time. Candidates could demonstrate competency by applying a relative credibility approach as well as other approaches such as correlation between increasing time lags.

In general, candidates either applied the relative credibility approach from Bailey and Simon or applied the correlation test from Mahler's "Shifting Risk Parameters". Application of these methods to the data was relatively straightforward, and several slightly different approaches were given credit.

A common mistake was using a strict actual versus expected, or variance approach, which does not directly address whether the underlying risk parameter is shifting over time.