1. (8.5 points)

An actuary has constructed a pure premium model using the Tweedie distribution with parameter $1 < p < 2$ to determine manual rates for a Workers' Compensation book of business. The output of the model is pure premium per $100 of payroll.

The following variables were considered for inclusion in the model:

Variable	Description or Source of Data	p-value
Industry	Construction, Manufacturing, All Other	0.002
Return to Work Program	Yes or No	0.008
Employee Age	Average Age in Years	0.003
Employee Tenure	Average Number of Years of Employment	0.005
Location	State of Jurisdiction	0.080
Employee Morale	Based on Results of Annual Company Survey	0.150
Number of Back Injuries	Supplied by Employer	0.010

The actuary has decided to use the following variables in the model: Industry, Employee Tenure, and Return to Work Program.

a. (1.5 point) Discuss the statistical and non-statistical considerations of including each of the three variables (Industry, Employee Tenure and Return to Work Program) in the model.

b. (2 points) Discuss the statistical and non-statistical considerations of excluding each of the remaining four variables (Employee Age, Location, Employee Morale and Number of Back Injuries) from the model.

c. (1.5 points) The actuary fits the log link GLM model using the three selected variables. Given the fitted model parameters below and the following information for a Manufacturing Workers' Compensation risk, calculate the standard premium for this risk for an annual policy period.

Parameter	Coefficient
Intercept	-0.631
Employee Tenure	-0.040
Return to Work Program: Yes	-0.200
Industry Type: Construction	0.350
Industry Type: All Other	-0.550

<<QUESTION 1 CONTINUED ON NEXT PAGE>>

Data for Manufacturing Risk	Value
Payroll	$1,000,000
Employee Tenure	5 Years
Return to Work Program	No
Actual Losses for Experience Rating	$12,500
Fixed Expenses	$1,500
Variable Expenses (as % of premium)	20%
Experience Rating Constant (K)	$10,000

d. (1.5 points) The actuary has graphed actual vs. modeled expected loss ratios by size of risk for the company's entire Workers' Compensation book, which is shown below.

In order to improve the model fit for larger risks, the actuary is considering incorporating the variable "Latest 3 Year Historical Losses" into the model. Explain three reasons against doing so.
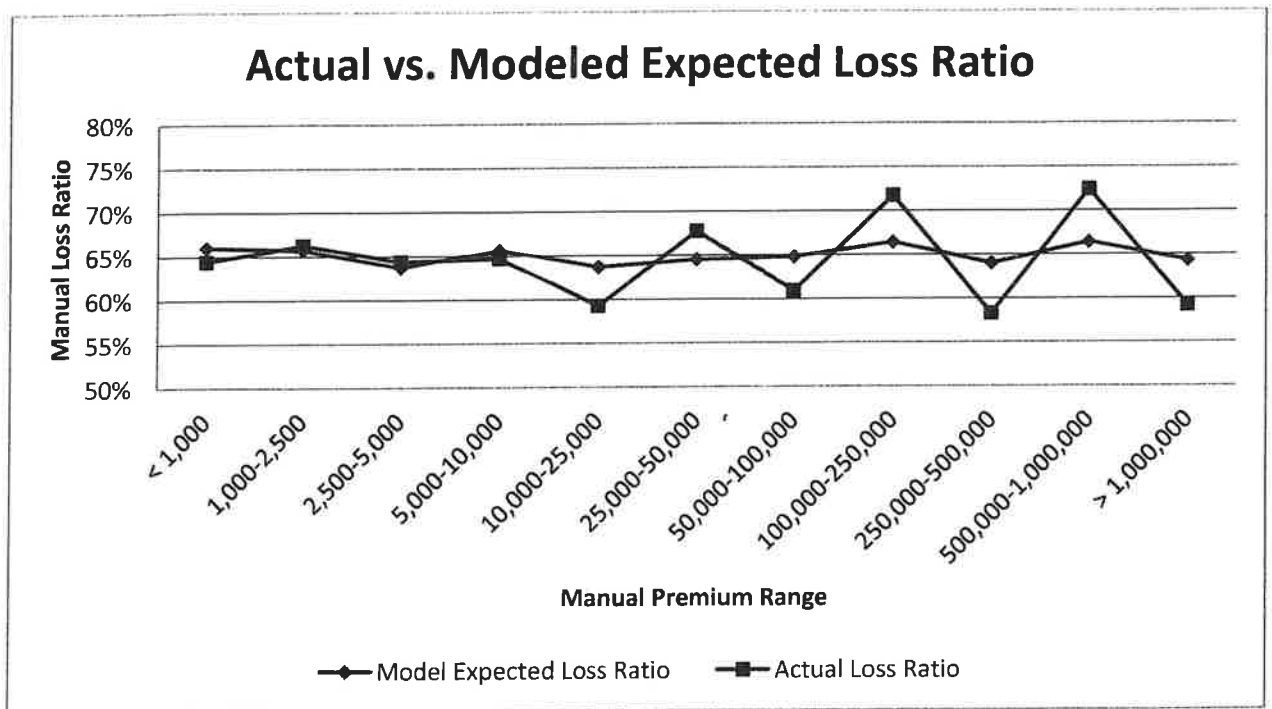
e. (2 points) The actuary is now developing a quote for a new Construction risk with $50,000,000 of payroll using this rating plan.

   i. (1 point) Describe two potential issues in developing a premium for this risk under this rating plan.

   ii. (1 point) Provide an alternative rating approach for this risk and briefly discuss the advantages of this alternative for the insured as well as the insurance company.

QUESTION 1	
TOTAL POINT VALUE: 8.5	**LEARNING OBJECTIVE(S): A1a, A1d, A3a, B3a, B3b, B5a, B6c**
SAMPLE ANSWERS	

Part a: 1.5 points

> **Industry** – industry has a low p-value indicating that this variable is statistically significant and a good predictor of future losses.
>
> Additionally, there is a connection between industry and overall loss levels (construction companies would be expected to have more losses than florists as an example)
>
> Industry is a variable that is practical and easy to collect and easy to verify (via SIC code)
>
> **Employee Tenure** – employee tenure has a low p-value indicating that this variable is statistically significant and a good predictor of future losses.
>
> Employee tenure is directly connected to future loss activity – new employees may not be as safe as employees who know the building, the machinery, or have experience with other loss control method (i.e. know how to set safety guards, etc.)
>
> Employee tenure can likely be easily collected by asking the customer or agent, but on the other hand some customers may not know it or may feel like it is easy to 'game'
>
> **Return to Work Program** – return to work program has a lower p-value indicating that this variable is statistically significant and a good predictor of future losses.
>
> There is a direct connection to the amount an employer spends on employee injuries – the quicker an employee returns to work, the less amount of time the employer needs to pay wage replacement. Additionally, this is an easy piece of information to obtain and since it is a discrete variable (yes or no) it should be easy to verify.

Part b: 2 points

> **Employee Age** – Employee age is has a lower p-value indicating that this variable is statistically significant and a good predictor of future losses.
> However, ASOP 3.2.5 discusses compliance with applicable law. Using age could be considered to be discriminatory and not in compliance with Federal laws.
> (Alternatively, some candidates argues that employee age could be correlated with employee tenure and should be excluded so that the model converges. Credit was given for this response.)
>
> **Location** – Location of the company would likely be an acceptable variable – companies that are located in areas with plenty of hospitals and doctors would likely have overall lower loss costs than companies in rural areas. However, the p-value for this variable is high indicating that it is not a good predictor of future losses.

Employee Morale – Employee morale does not appear to be predictive of future losses based on the high p-value being generated from the model. Furthermore, the variable is very subjective, difficult to collect and difficult to verify so it also does not meet the standards set in ASOP 12.

Number of Back Injury Claims - # of back injury claims has a relatively small p-value so it may have some predictive power. However, it fails many of the ASOP 12 considerations including causality – it's not clear that the # of back injuries is directly related to losses – injuries could be small or large. Additionally, since the information is supplied by the employer, it may be tough to verify and be subject to gaming.

Part c: 1.5 points

There are five different elements where there are two valid assumptions for each one:
  i.   Candidates could interpret Manufacturing to be part of All Other Industry type OR the base class
  ii.   Candidates could take the Employee Tenure variable either directly or by taking the natural log
  iii.   Candidates could assume the given actual experience includes one year of experience OR three years of experience
  iv.   Candidates could apply the experience modification factor either before or after expense loading
  v.   Candidates could calculate the experience modification factor directly or using a credibility formula. These two approaches are numerically identical.

The following are a sample of acceptable full credit approaches

Sample 1 – Using Manufacturing as the base industry class / actual experience includes 1 year / employee tenure is not transformed / experience mod calculated directly / experience mod applied after expense load

$\ln(u) = -0.631 + (-.040 * 5) + (-0.20 * 0) + (-.55 * 1)$
$\ln(u) = -1.381$
$u = .251$ or $0.251 per $100 of

Manual Premium =
$0.251 * ($1,000,000/$100) = $2,513

P = Losses + Fixed Expenses + Variable Expenses
P = $2,513 + $1,500 + 0.2P
0.8P = $4,013
P = $5,017

M = (A + K) / (E + K)
= ($12,500 + $10,000) / ($2,513 + $10,000) = $22,500 / $12,513
=1.798

Standard Premium = $5,017 * 1.798 = $9,020

*Sample 2 - **Using Manufacturing as the All Other industry class** / actual experience includes 1 year / employee tenure is not transformed / experience mod calculated directly / experience mod applied after expense load*

ln(u) = -0.631 + (-.040*5) + (-0.20 * 0)

ln(u) = -0.831

u = .436 or $0.436 per $100 of

Manual Premium =

$0.436 * ($1,000,000/$100) = $4,356

P = Losses + Fixed Expenses + Variable Expenses

P = $4,356 + $1,500 + 0.2P

0.8P = $5,856

P = $7,320

M = (A + K) / (E + K)

= ($12,500 + $10,000) / ($4,356 + $10,000) = $22,500 / $12,513

=1.567

Standard Premium = $7,320 * 1.567 = $11,470

*Sample 3 - Using Manufacturing as the base industry class / actual experience includes 1 year / **employee tenure IS transformed** / experience mod calculated directly / experience mod applied after expense load*

u = -0.631 + -.040*ln(5) + (-0.20 * 0)

ln(u) = -0.6954

u = .4989 or $0.4989 per $100 of

Manual Premium =

$0.4989 * ($1,000,000/$100) = $4,989

P = Losses + Fixed Expenses + Variable Expenses

P = $4,989 + $1,500 + 0.2P

0.8P = $6,489

P = $8,111

M = (A + K) / (E + K)

= ($12,500 + $10,000) / ($4,989 + $10,000) = $22,500 / $14,989

=1.501

Standard Premium = $8,111 * 1.501 = $12,175

*Sample 4 - Using Manufacturing as the base industry class / **actual experience includes 3 years** / employee tenure is not transformed / **experience mod calculated via a credibility approach** / experience mod applied after expense load*

$\ln(u) = -0.631 + (-.040*5) + (-0.20 * 0)$

$\ln(u) = -0.831$

$u = .436$ or $0.436 per $100 of

Manual Premium =

$0.436 * ($1,000,000/$100) = $4,356

P = Losses + Fixed Expenses + Variable Expenses

P = $4,356 + $1,500 + 0.2P

0.8P = $5,856

P = $7,320

E = 4356*3 = 13,068

Z = 13,068/(13,068 + 10,000) = .566

E Mod = (12,500 * .566 + 13,068 * (1 - .566))/13,068 = .975

$7,320 * .975 = $7,137

Part d: 1.5 points

Candidates responding with any three of the following would receive full credit:
- Historical loss experience already impacts premium through experience rating. They should not be double considered.
- "Latest 3 year losses" is highly correlated to premium range. If we want to use historical loss experience we'd better use loss ratio.
- The bad fit on the higher range might be caused by sparse data of large premium insureds. So it is normal and maybe how much effort to improve fit can be a waste of time
- Small risks have volatile experience and incorporating latest 3 years historical losses will cause fluctuations in their premium
- This variable will most likely be highly correlated with other variables which may lead to an unstable model. If multicolinearity is present, the model may not converge at all or lead to irrational outputs.
- The latest 3 yr losses won't be fully developed. So, if the reporting lag varies by company it would be over or under predicting the loss experience.

- The variable would be very correlated with risk size since larger insureds are more likely to have higher historical losses so including the variable without stating it as a ratio to exposure, manual premium or expected loss could skew results by size.
- It would be unfair as not all losses are predictive of future – for example random large losses – they should be capped and credibility weighted with expected.
- 3 year historical losses does not separate frequency from severity. May be better to build variables that separate these impacts.
- There may be recent changes to the risk's safety procedures or WC benefit levels that would not yet be fully reflected in 3 yr historical loss.

Part e: 2 points

Sample Responses for part i)

- Since risk is new, there won't be any actual loses with which to do experience rating, so it is difficult for insurer to know whether manual premium is adequate and not excessive for this risk. (new – no loss)
- High payroll amount puts the risk in the high manual premium range, where model is less successful at predicting the actual losses – so there's additional uncertainty in manual premium due to this. (large fit)
- Based on the graph in part C, I assume that the experience shown in the larger premium range is thin so the company may not have enough data to feel confident on pricing risks of this size. (thin)
- The experience rating constant K is likely too low for this risk as it is giving very high credibility to actual experience of a risk of $50M. (k low)
- The experience rating plan does not seem to have a cap or apply a split between primary and excess losses which may lead to oversensitivity to large loss events. (no split)
- Large companies may have more effective return to work programs or other unique characteristics that are not contemplated by this model that doesn't consider variables for large size risks separately (large unique)
- The variable expenses of 20% may be appropriate for smaller risks but could be too high for a risk of this size and the plan does not include any expense discount to account for this. (expenses)
- Having only one group in GLM for all construction likely insufficiently addresses differences in WC loss potential between different hazard levels of construction work such as roofing vs. drywall installation.
- Assuming the question implies rate is set only using GLM without experience mod, then the risk has little incentive to control losses.

Sample Responses for part ii)

- They should enter in to a retro rating agreement. Since the new risk has no prior experience to base price on, the retro can allow adjustments to premium based on current period experience. For the insured, the retro plan may result in cash flow advantages if the initial premium is set lower. For the insurer, one advantage is a lower capital requirement.
- A large deductible plan could be considered as a loss sensitive rating option where the insured is responsible for claims below the deductible. Benefit to the insured: Would see

a more immediate reflection of good loss experience. Benefit to the insurer: the insured has a financial incentive for loss control.

- Base the experience rating component of pricing on the NCCI method of splitting primary and excess losses so that premium is not inappropriately impacted by large losses with less credibility. The advantage for the insured is that their premium will not see large swings year to year from large loss experience. The advantage for the insurer is that the experience mods from a split plan for WC are generally shown to be more accurate in estimating expected costs so they are less likely to underprice risk.
- A new GLM could be developed to estimate pure premium that focuses on variables more relevant to construction risks. This could remove some of the premium uncertainly for larger construction policies. For the insurer, the model will provide more insights into drivers of expected loss. For the insured, the premium would be more likely to appropriately reflect their existing characteristics and therefore be more equitable.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of both statistical and non-statistical considerations for the inclusion or exclusion of variables in a model. Candidates were also expected to demonstrate knowledge of experience rating along with their knowledge of GLMs.

Part a

Candidates were expected to provide statistical and non-statistical reasons for including the given variables in the model. Additional responses that related back to the Actuarial Standards of Practice were also accepted for the non-statistical responses.

Common mistakes included:
- Omitting either a statistical or non-statistical consideration for any of the variables
- Not arguing for inclusion in the model

Part b

Candidates were expected to provide statistical and non-statistical reasons for excluding the given variables in the model. Additional responses that related back to the Actuarial Standards of Practice were also accepted for the non-statistical responses.

Common mistakes included:
- Omitting either a statistical or non-statistical consideration for any of the variables
- Not arguing for exclusion from the model

Part c

Candidates were expected to calculate a standard premium based on model parameters and other data provided. There were many possible answers that received full credit based on different interpretations of several elements to this question. Any correctly calculated answer based on the varying selected assumptions in i. through iv. below received full credit.

  i. Candidates could interpret Manufacturing to be part of All Other Industry type OR the base class
  ii. Candidates could take the Employee Tenure variable either directly or by taking the natural log

iii.	Candidates could assume the given actual experience includes one year of experience OR three years of experience
iv.	Candidates could apply the experience modification factor either before or after expense loading
v.	Candidates could calculate the experience modification factor directly or using a credibility formula. These two approaches are numerically identical.

Common mistakes included:
- Not calculating an experience mod at all
- Calculation errors

Part d

From LO/KS B3a, candidates were expected to be able to draw on their knowledge of experience rating for WC and recognize that the standard premium calculation would include the application of an experience mod based on recent loss experience which would have some overlap with the experience rating plan leading to 'double counting' of experience. Furthermore, the candidate should be able to pick up some of the following reasons against using total three year losses directly in the model:

- Use of total losses could result in volatile pricing outcomes from large loss events
- For smaller risks, 3 year losses may not be predictive of future outcomes since their year-to-year actual results are expected to have more variation than large risks
- Losses should be scaled by exposure or expected loss otherwise the correlation of total loss with manual premium will distort the fit
- To provide an appropriate prediction for future loss, past 3 year losses may need to be developed and adjusted for changes in benefit levels or changes in the risk's size, operations, or safety practices over the three year period vs. prospective period

Although the question was designed to focus on concepts of experience rating, alternative responses related to knowledge of GLMs based on LO 4 were also accepted. This could include:

- Adding more parameters or degrees of freedom to the model could lead to over-fitting
- Correlation between variables in a model can lead to erratic coefficients and the prior loss could be highly correlated with in-model predictors
- In practical application it is typical when fitting commercial models for there to be fewer risks in the large premium bucket so the lack of fit may still be within acceptable variation for actual results

Credit was not awarded for the following types of responses:

- Comments that correlation of variable with target were given no credit because we are trying to find variables that are correlated with the target.
- Comments that 3 year historical losses lack credibility or that GLM gives full credibility were not given credit unless they included specifics about large losses or small risks.
- Comments that large loss experience was volatile were not given credit – the lack of fit in the higher range does not necessarily imply more volatile experience for larger risks – more likely there are risk differences not captured by the GLM.

- Comments that there could be difficulty collecting prior 3 year losses were not given credit because it was already given in part c of the question that they would be collected for experience rating anyway.
- Comments that it is not industry practice to include prior losses in a GLM or that there might not be public acceptance or regulatory approval were not given credit because this is something that is commonly done in the industry.
- Comments related to appropriateness of use of Tweedie model were not given credit because although the reading states that Tweedie assumptions are violated when frequency and severity do not move together, it is very common practice for commercial pricing models with a wide array of frequency and severity profiles to use Tweedie models. Note the reading also states, "However, Tweedie GLMs can be quite robust against such violations of its assumptions and still produce very strong models." ("Generalized Linear Models for Insurance Rating," CAS Monograph #5, pg 23)

Common mistakes included:
- Interpreting the chart as a quintiles test output or lift chart, not a graph of model fit by premium range or, similarly, thinking that the groups on the X axis of the chart were predicted loss ratios, not manual premium ranges
- Believing that loss ratios should be either monotonically increasing or decreasing by size of risk, when in reality if the prediction is fairly good they should be fairly flat other than not capturing experience component that is captured in standard premium.

Part e

Candidates were expected to recognize that the fit of the pricing model for larger risks was not doing a good job of predicting losses for larger accounts so the premium could be too high or too low. Additionally, candidates were expected to recognize that there would be some sort of issue with experience rating this risk, either making the assumption that "new Construction Risk" indicated a new operation with no prior loss experience or recognizing that the experience rating plan was offering too much credibility or should split excess vs. primary in some way. However, assuming that the risk was only new to the company could also result in full credit answers.

To get full credit a response needed to include sufficient description of the issue as in the sample responses given above. Responses that briefly identified an acceptable issue but did not describe it only received partial credit.

Credit was not given for responses that said larger risks should be experience rated and not rely on manual rating – in part (c) of the problem we are told that manual premium is subject to experience rating to get to standard premium in this pricing plan.

Credit was not given for responses stating that variables other than prior loss experience would be unknown or cause problems for a new risk. It is expected that candidates should assume underwriters would evaluate the risk and collect data required for manual rating.