13. (2.5 points)

A risk is written using a balanced retrospective rating plan with the following characteristics:

- Losses at the minimum premium = $50,000
- Losses at the maximum premium = $300,000
- Loss conversion factor = 1.05
- e = $10,000

The following table shows actual experience from a representative sample of risks that are similar to the risk in question:

Risk	Actual Aggregate Loss	Risk	Actual Aggregate Loss
1	$25,000	6	$175,000
2	$50,000	7	$200,000
3	$100,000	8	$300,000
4	$100,000	9	$350,000
5	$150,000	10	$550,000

a. (2 points)

Determine the maximum premium that the insured can be charged.

b. (0.5 point)

An actuary has acquired actual aggregate data from a new book consisting of risks within the same industry. The loss experience of five representative risks from this book is shown below:

Risk	Actual Aggregate Loss
1	$275,000
2	$300,000
3	$500,000
4	$700,000
5	$800,000

The actuary proposes to combine the data from the two books, stating that the combined data will result in a more accurate calculation of the insured's retrospective premium. Assess the validity of the actuary's statement.

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QUESTION 13	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): B5a**
SAMPLE ANSWERS	

Part a: 2.0 points

Sample 1

E[A]= (25 + 50 + 100 + 100 + 150 + 175 + 200 + 300 + 350 + 550) / 10 = 200K

A	r	# Risks	# Risks over r	% Risks over r	ϕ(r)
0	0.0	0	10	100%	1.0
25	0.125	1	9	90%	0.875
50	0.25	1	8	80%	0.7625
100	0.5	2	6	60%	0.5625
150	0.75	1	5	50%	0.4125
175	0.875	1	4	40%	0.35
200	1.0	1	3	30%	0.3
300	1.5	1	2	20%	0.15
350	1.75	1	1	10%	0.1
550	2.75	1	0	0%	0.0

$\phi(0.25) = 0.7625$

$\psi(0.25) = 0.7625 + 0.25 - 1 = 0.0125$

$\phi(1.5) = 0.15$

$b = e - (c\text{-}1)E[A] + cI$

$I = E[A](\phi(1.5) - \psi(0.25))$

$b = 10K - (1.05 - 1)*200K + 1.05*(200K)*(.15 - 0.0125) = 28{,}875$

$G = (b + cL_G)$

$G = 28{,}875 + 1.05*300{,}000 = 343{,}875$

Sample 2

E(A) = 200,000

$L_H = r_H * E(A) \rightarrow r_H = 0.25$

$L_G = r_G * E(A) \rightarrow r_G = 1.5$

Construct Table M:

r	ϕ(r)
0.125	0.875
0.25	0.7625
0.5	0.5625
0.75	0.4125
0.875	0.35
1.0	0.3
1.5	0.15
1.75	0.1
2.75	0.0

ϕ(1.5) = 0.15

ϕ(0.25) = 0.7625

Assume Tax multiplier = 1

$\phi(r_H) - \phi(r_G) = [(e + E[A]) - H] / (C*E[A]*T)$

0.7625 – 0.15 = (210,000 – H) / 210,000

H = 81,375

$r_G - r_H = [G - H] / (C*E[A]*T)$

1.5 – 0.25 = (G – 81,375) / (1.05*200,000)

G = 343,875

Sample 3

E[A]= (25 + 50 + 100 + 100 + 150 + 175 + 200 + 300 + 350 + 550) / 10 = 200,000

Charge = ((350,000 – 300,000) + (550,000 – 300,000))/10 = 30,000

Savings = (50,000 – 25,000) /10 = 2,500

I = 30,000 – 2,500 = 27,500

$b = e – (c – 1)*E + cI$

$b = 10,000 – (1.05 – 1)*200,000 + 1.05*27,500 = 28,875$

Assume no prem tax:

$G = b + cL_G$

$G = 28,875 + 1.05*300,000 = 343,875$

Part b: 0.5 point

Sample 1

E[A] from sample in part (a) = 200,000.
E[A] from 5 new risks in part (b) = 515,000

The actuary is incorrect. You should combine if the expected value of losses and the loss distribution are the same. In this case, the new risks E[A] is significantly larger than that of part (a).

They could possibly have a similar distribution since coming from same industry; however since the expected loss is so much larger, they should be using different table Ms.

Sample 2

The new book of business has an aggregate loss distribution with much higher losses than the current book. A separate loss curve and separate Table M should be used. The new book is not reflective of the old book of business and combining them will significantly affect risk charges leading to inaccurate premiums that are not reflective of the distribution of potential losses.

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT

The candidate was expected to use the concepts of Table M insurance charges and savings to construct a loss sensitive rating plan, and to understand the actuarial principles and concepts underlying the construction of a retrospective rating plan. Overall, candidates performed well on this question.

Part a

Candidates were expected to correctly calculate the parameters of a retrospective rating plan to find the maximum premium under the plan.

If a candidate stated an assumption regarding a tax rate that would have resulted in a different answer from the sample solutions, credit was awarded provided the assumption was applied properly.

Common mistakes included:

- Using incorrect formulas for the basic premium and/or maximum premium
- Using expected limited loss, instead of total expected loss
- Failing to calculate the net insurance charge on a per risk basis (not averaging the charge and/or savings across all risks)

Part b

Candidates were expected to demonstrate the understanding that Table M should be calculated using experience from risks of a similar size and subject to similar hazards, and to assess the validity of the actuary's statement.

Common mistakes included:

- Not providing an assessment of the actuary's statement
- Failing to state, in some form, that creating a combined Table M using risks with different aggregate loss distributions in the same Table M is not appropriate