14. (3.0 points)

A reinsurer has been supplied the following information from a large insurance company:

Claim Size Range	Expected Number of Claims	Expected Ultimate Losses (000s)
$0 to $1,000,000	19,000	$6,750,500
$1,000,001 to $2,000,000	359	$525,300
$2,000,001 to $3,000,000	230	$566,500
$3,000,001 to $4,000,000	147	$507,700
above $4,000,001	264	$1,650,000
TOTAL	**20,000**	**$10,000,000**

The reinsurer is entering into an excess of loss contract with the primary insurance company. The reinsurer will pay all losses above a $5,000,000 per claim retention.

a. (1.5 points)

Construct a graph of the excess severity function for claim sizes of $1,000,000, $2,000,000, $3,000,000, and $4,000,000.

b. (1.5 points)

Calculate the reinsurer's expected losses under the proposed contract.

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QUESTION 14

TOTAL POINT VALUE: 3.0 | **LEARNING OBJECTIVE(S): B1d**

SAMPLE ANSWERS

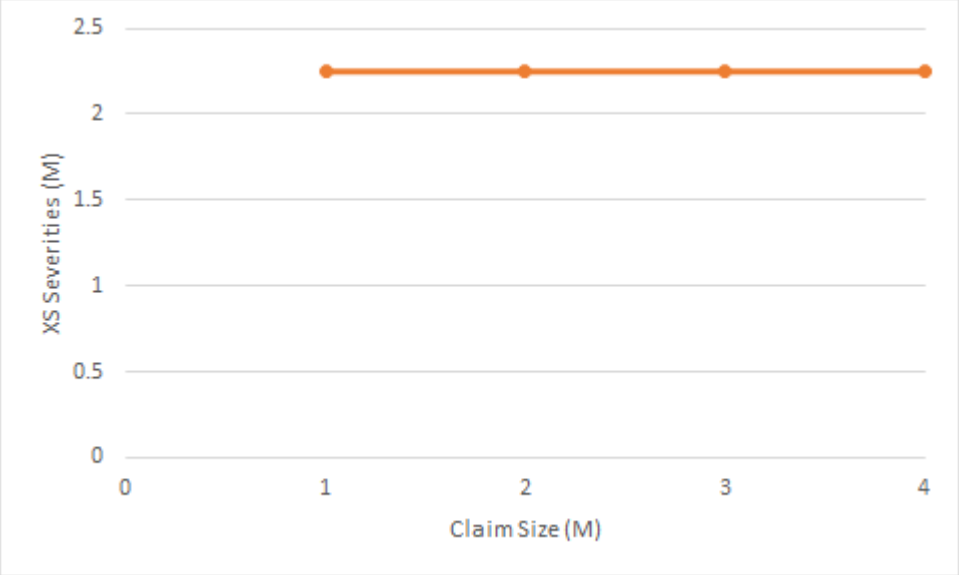
Part a: 1.5 points

Sample 1

Limit, L	1-F(L)	E[X;L]	XS Severity at L
1,000,000	.05	387,525	2,249,500
2,000,000	.03205	427,890	2,249,922
3,000,000	.02055	453,765	2,249,878
4,000,000	.0132	470,300	2,250,000

E[X] = 10,000,000,000/20,000 = 500,000
E[X; 1M] = (6,750,500,000 + 1000(1,000,000))/20,000 = 387,525
E[X; 2M] = (6,750,500,000 + 525,300,000 + 2,000,000(230 + 147 + 264))/20,000 = 427,890
E[X; 3M] = (6,750,500,000 + 525,300,000 + 566,500,000 + 3,000,000(147 + 264))/20,000 = 453,765
E[X; 4M] = (6750,500,000 + 525,300,000 + 566,500,000 + 507,700,000 + 264(4,000,000))/20,000 = 470,300

XS Severity at L = (E[X] – E[X; L])/(1-F(L))

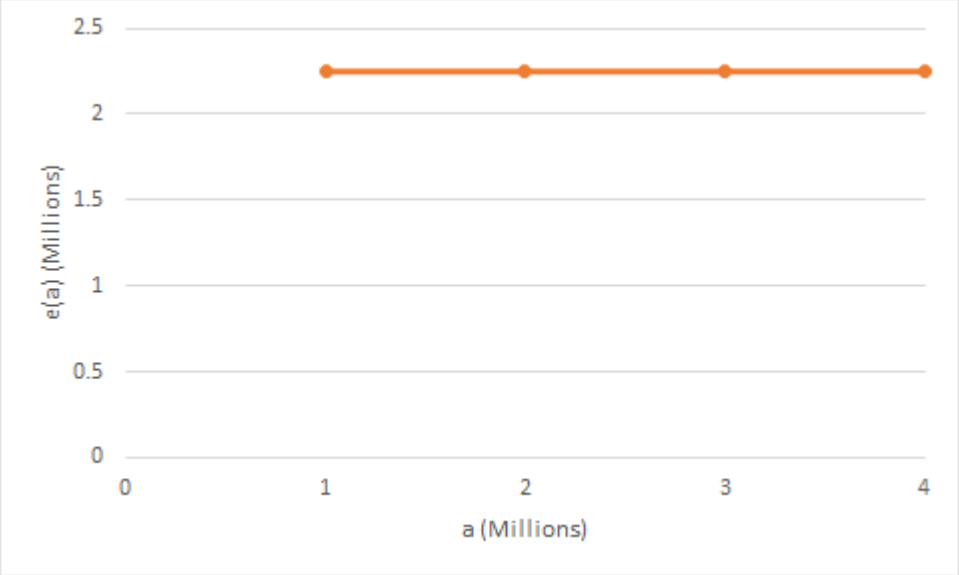


Sample 2

Excess sev = e(a) = Sum(loss above a – a * claims above a)/# above

e(1M) = (10M – 6,750,500 – 1000 * (359 + 230 + 147 + 264))/(359 + 230 + 147 + 264) = 2,249.5k
e(2M) = (566,600 + 507,700 + 1,650,000 – 2000 * (230 + 147 + 264))/(230 + 147 + 264) = 2,249.9k

e(3M) = (507,700 + 1,650,000 – 3000 * (147 + 264))/(147 + 264) = 2,249.9k
e(4M) = (1,650,000 – 4000 * 264)/264 = 2,250k



Part b: 1.5 points

Sample 1

From a. excess severity is flat

- ➔ Losses follow exponential distribution above 1M with beta = 2.25M
- ➔ e(5M) = e(4M) = … = e(1M) = 2.25M
- ➔ $P(X>5M|X>1M) = e^{-(5-1)M/2.25M} = 0.169$
- ➔ E[N|X>5M] = (20k – 19k) * 0.169 = 169.01
- ➔ Reinsurer E[L] = 169.01 * 2.25M = 380.28M

Sample 2

(1)	(2)	(3)	(4) = (3)/(2)
Limit (L)	Claims over L	Claims over L + 1M	% of claims in L that are over L + 1M
1	20k – 19k = 1000	1000 – 359 = 641	641/1000 = 0.64
2	641	641 – 230 = 411	411/614 = 0.641
3	411	411 – 147 = 264	264/411 = 0.642
4	264	264 * .641 = 169.22	.641 <- selected based on constant pattern

Based on memoryless feature of exponential distribution, we estimate that there are $E[N_{5M}]$ = 169.22 claims above 5M. From before, we estimated that $E[X_{5M}]$ = 2.25M

So we estimate reinsurer expected loss as $E[S] = E[X_{5M}]*E[N_{5M}]$ = 2.25M * 169.22 = 380,745,000

EXAMINER'S REPORT

Candidates were expected to calculate excess severities given claims data at various loss layers, determine the underlying distribution from the excess severity pattern, and calculate total expected losses in a given loss layer.

Candidates generally performed well on part (a). Common reasons for not receiving full credit on part (a) included calculating and graphing the incorrect metric, misinterpreting claims information provided as layer losses instead of ground up, and using an incorrect formula for excess severity.

Part (b) was generally difficult for candidates. Common reasons for not receiving full credit on part (b) included:

- Assuming the exponential distribution applied to ground up loss instead of loss excess 1M
- Assuming all losses above 4M also exceeded 5M

This question was based on Chapter 5 of the Bahnemann text.

Part a

Candidates were expected to calculate the excess severities above each of the indicated limits given the loss data provided and to graph them with properly labeled axes.

Candidates who calculated and plotted the incorrect metric, such as the CDF or excess ratio, received no credit as this did not demonstrate knowledge of what an excess severity was.

Common mistakes included misinterpreting the claims information provided as layer losses instead of ground up, or using an incorrect formula for excess severity (such as calculating ground up severity instead of excess severity for claims that crossed a given limit). The former is an incorrect assumption because even if a candidate misinterpreted the table, the severities produced by this method would result in layer severities greater than the 1M layer (ex: 525,300,000/359 = 1.46M average which is > the 1M layer span).

Candidates who incorrectly calculated excess severities but correctly plotted and labelled them received partial credit.

Part b

Candidates were expected to correctly identify the underlying loss distribution given the graphed excess severities and apply it to calculating the total expected excess losses above 5M.

Candidates who assumed all losses and claims above 4M were also above 5M did not receive full credit as this was not an assumption that could be drawn from the data provided.

Common mistakes included:

- Not identifying a distribution from the graphed excess severities
- Assuming the exponential distribution applied to ground up loss instead of loss excess 1M
- Using the overall ground up mean as the beta parameter for the exponential distribution instead of 2.25M

Candidates who identified a correct distribution and inferred a reasonable excess severity in part (b) based off of incorrectly calculated excess severities from part (a) received partial credit.

Some candidates identified a Pareto distribution and then calculated a constant rate of decrease in the survival function to get number of claims excess of 5M. As a constant rate of decrease is a property of an exponential function and not a Pareto, only partial credit was provided.