1. (9 points)

An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
Accident Years	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	$12,033,000	$1,766,000	$2,329,000	$1,236,000	170	170
1996-2000	$13,812,000	$1,819,000	$2,762,000	$1,273,000	210	172
2001-2005	$13,985,000	$1,751,000	$2,797,000	$1,506,000	210	201
2006-2010	$16,444,000	$1,710,000	$3,288,000	$1,471,000	240	195
2011-2015	$17,507,000	$1,673,000	$3,350,000	$1,439,000	250	198
Total	$73,781,000	$8,719,000	$14,526,000	$6,925,000	1,080	936

a. (1.5 points)

Conduct chi-square tests with an α value of 0.10 on actual vs. expected claim counts to confirm whether or not risk parameters have shifted over time. Use the following table of critical values:

Degrees of Freedom	Critical Value $\alpha = 0.10$
1	2.706
2	4.605
3	6.251
4	7.779
5	9.236
6	10.645

b. (1.5 points)

To select an expected future claim frequency for LOB 2, the actuarial consultant has decided to assign equal weight (Z/2) to each of the most recent two groups of accident years and the remaining weight (1-Z) to the overall mean frequency.

Calculate the expected future claim frequency for LOB 2 by first using the mean-squared-error (MSE) criterion to determine the optimal value for Z from the following three choices:

Z value	MSE
0.10	Not provided
0.50	0.0190%
0.90	0.0164%

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(The following information relates to parts c., d., e., f., and g. below)

The insurance company is planning to write $10,000,000 of new business for LOB 1 in the new territory. The following reinsurance treaty options are available to the insurance company to support its expansion:

Option 1: 2,000,000 xs 6,000,000 Aggregate Excess of Loss
- Rate on Line = 12.5%

Option 2: 25% Aggregate Quota Share
- Ceding Commission = 20%
- Target ceded profit of 20%, which specifies that if ceded premiums less ceding commission and ceded loss exceeds 20% of ceded premiums, the excess is paid back to the cedent through a profit commission
- Aggregate Ceded Loss Ratio Cap = 220%

c. (2.25 points)

Calculate the ceded profit for each reinsurance option for the following gross loss amount scenarios:

Scenario	LOB 1 Gross Loss Amount
1	5,000,000
2	10,000,000
3	20,000,000
4	30,000,000

d. (1.5 points)

The actuarial consultant has decided that a prospective loss ratio of 20% for LOB 1 is appropriate, but that the gross capital required to support the expansion is $8,000,000 based on the insurance company's requirement to hold capital at a 1-in-100 year return period. Assume that aggregate LOB 1 ground-up losses are lognormally distributed.

The cumulative distribution function of a lognormal distribution is: $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$, where Φ is the standard normal CDF, and μ and σ are the mean and standard deviation of the associated normal distribution.

The mean and variance of a lognormal distribution are given below, as well as a table of values from the standard normal distribution function:

$$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$Var(X) = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$$

x	Φ(x)
1.751	96.00%
1.814	96.52%
1.881	97.00%
1.916	97.23%
2.054	98.00%
2.121	98.31%
2.326	99.00%
2.531	99.43%

Determine the coefficient of variation for LOB 1, assuming it is less than 100%.

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e. (1 point)

Calculate the probability of each of the following for LOB 1 assuming aggregate losses are lognormally distributed:

i. Attaching the 2,000,000 xs 6,000,000 Aggregate Excess of Loss treaty
ii. A negative ceded profit on the Aggregate Quota Share

f. (0.5 point)

State one advantage and one disadvantage of the Aggregate Quote Share from the perspective of the reinsurer.

g. (0.75 point)

Recommend and justify which reinsurance treaty option the insurance company should select for LOB 1. Assume both reinsurance treaty options provide the same amount of capital relief.

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QUESTION 1

TOTAL POINT VALUE: 9 | **LEARNING OBJECTIVE(S): A1, B1, B2, C3, C4**

SAMPLE ANSWERS

Part a: 1.5 points

Sample 1

D.O.F = n-1 = 5 – 1 = 4
Critical Value = 7.779

$$\chi^2 = \sum \frac{(A-E)^2}{E}$$

Avg Freq LOB 1: 1080/73781 = 0.0146
Avg Freq LOB 2: 936/8719 = 0.1074

Accident Years	LOB 1 Actual	LOB 1 Expected	LOB 2 Actual	LOB 2 Expected
1991-1995	170	176.14	170	189.58
1996-2000	210	202.18	172	195.27
2001-2005	210	204.71	201	167.97
2006-2010	240	240.71	195	183.57
2011-2015	250	256.27	198	179.60
Chi-Squared		0.809		8.295

LOB 1 H_O: Risk parameters have not shifted over time
0.809 < 7.779
LOB 1 count parameters have not shifted, accept null hypothesis

LOB 2 H_O: Risk parameters have not shifted over time
8.295 > 7.779
Reject null hypothesis, parameters have shifted as the χ^2 statistic is greater than the threshold.

Sample 2:

Since exposure is not given, use prem as exposure base

Accident Years	Freq LOB 1	Freq LOB 2
1991-1995	170/12033 = 0.01413	170/1766 = 0.09626
1996-2000	0.01520	0.09456
2001-2005	0.01502	0.11479
2006-2010	0.01459	0.11404
2011-2015	0.01428	0.11835

Expected freq assumed to be the overall mean:
LOB1 -> 1080 / 73781 = 0.01464
LOB2 -> 936 / 8719 = 0.10735

LOB1:

$$\chi^2 = \sum \frac{w(expected - actual)^2}{expected}$$

$$\chi^2 = \frac{12033(0.01413-0.01464)^2+\cdots+17507(0.01428-0.01464)^2}{0.01464}$$

= 0.8084 < 7.779
Therefore no shift over time

LOB2:

$$\chi^2 = \sum \frac{w(expected - actual)^2}{expected}$$

$$\chi^2 = \frac{1766(0.09626-0.1074)^2+\cdots+1673(0.1184-0.1074)^2}{0.1074}$$

= 8.296 > 7.779
Therefore there is a shift in risk parameters over time.

Part b: 1.5 points

Sample 1

195/1710 = .1140 Freq/1000 prem 2006 – 2010
198/1673 = .1184 Freq/1000 prem 2011 – 2015
936/8719 = .1074 All Years

170/1766 = .0963 (1991-1995)
172/1819 = .0946 (1996-2000)
201/1751 = .1148 (2001-2005)

For Z=0.1:

(.05)(.0963) + (.05)(.0946) + (.9)$\left(\frac{170+172}{1766+1819}\right)$ = .0954 expected 2001-2005

(.05)(.1148) + (.05)(.0946) + (.9)$\left(\frac{170+172+201}{1766+1819+1751}\right)$ = .1021 expected 2006-2010

(.05)(.1140) + (.05)(.1148) + (.9)$\left(\frac{170+172+201+195}{1766+1819+1751+1710}\right)$ = .1057 expected 2011-2015

$((.1148 - .0954)^2 + (.1140 - .1021)^2 + (.1184 - .1057)^2)/3 = 0.00022642$
=.0226% = MSE for Z = 0.1

MSE is lowest for Z = 0.9, so we use that selection.

Expected Freq:
(.45)(.1140) + (.45)(.1184) + (.1) (.1074) = .1153

Sample 2

Test Z = 10% - choosing to only test the most recent 3 years groups since they have 2 prior groups each

AY	Actual Freq	Expected Freq w/Z=0.1
01-05	201/1751M=0.000115	(.05) $\left(\frac{170}{1766M}\right)$+ (.05) $\left(\frac{172}{1819M}\right)$+(.9)$\left(\frac{170+172}{1766M+1819M}\right)$=0.0000954
06-10	0.000114	0.0001021
11-15	0.000118	0.0001057

Overall freq = 936/8719000 = 0.000107

$$MSE = \sum \frac{(Actual - Exp)^2}{3}$$
$$= 2.3 \times 10^{-10}$$

So we use Z=0.1, since it has the lowest MSE

Then expected frequency is projected = 0.9(0.000107) + 0.05(198/1673000) + 0.05(195/1710000) =0.000108 claims/earned premium

Part c: 2.25 points

Sample 1

Option 1:
Ceded premium = 12.5% × 2,000,000 = $250,000

Scenario	Gross Loss	Ceded Premium	Ceded Loss	Ceded Profit
1	5,000,000	250,000	0	250,000
2	10,000,000	250,000	2,000,000	-1,750,000
3	20,000,000	250,000	2,000,000	-1,750,000
4	30,000,000	250,000	2,000,000	-1,750,000

Option 2:
Ceded Premium = 25% (10,000,000) = 2,500,000
Ceding Commission = 20% (2,500,000) = 500,000

Scenario	Gross Loss	Ceded Loss	Ceded Premium	Ceding Commission	Profit Commission
1	5,000,000	25% (5,000,000) =1,250,000	2,500,000	500,000	250,000
2	10,000,000	2,500,000	2,500,000	500,000	0
3	20,000,000	5,000,000	2,500,000	500,000	0
4	30,000,000	220%(2,500,000) =5,500,000	2,500,000	500,000	0

Scenario	Ceded Profit
1	2,500,000-500,000-250,000-1,250,000 = 500,000
2	2,500,000-500,000-2,500,000 = -500,000
3	2,500,000-500,000-5,000,000 = -3,000,000
4	2,500,000-500,000-5,500,000 = -3,500,000

Sample 2

Scenario 1: Gross loss $5M
Option 1:
Ceded Premium = 2M × 12.5% = 250,000
Ceded loss = 0 => Profit = 250,000
Option 2:
Ceded premium = 10M × 25% = 2,500,000
Ceded Loss = 5M × 25% = 1,250,000 => Ceded LR = 50%
(Premium – Loss - Commission) / Premium = 1 - 0.5 – 0.2 = 30% > 20%
=> Ceded Profit = 1 - 0.50 – 0.20 – 0.10 = 20%, or 20% × 2.5M = 500,000

Scenario 2: Gross loss $10M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 2. 5M => Ceded LR = 100%
=> Profit = 1-1-.2 = -20%, or -500,000

Scenario 3: Gross loss $20M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 20×.25 = 5M => Ceded LR = 5M/2.5M = 200%
=> Profit = 1-2-.2 = -120%, or -3.0M

Scenario 4: Gross loss $30M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 30×.25 = 7.5M => Ceded LR = 7.5M/2.5M = 300% > Ceded LR Cap
⇨ Ceded LR = 220% => Profit = 1-2.2-.2 = -140%, or -3.5M

Sample 3

Option 1:
Premium = $2M × 12.5% = 250,000
Scenario 1: Ceded loss = 0 Ceded profit = 100%
Scenario 2: Ceded loss = 2M " " = 1 – 2,000,000/250,000 = -700%
Scenario 3: " " = 2M " " = -700%
Scenario 4: " " = 2M " " = -700%

Option 2:
Ceded Premium = 10M × 0.25 = 2,500,000
Max Ceded = 2.5 × 220% = 5.5M

Scenario 1: Ceded loss = 5M × 0.25 = 1.25M
Ceded profit = min(20%, (2.5 × (1-20%)-1.25)/2.5M) = 20%
Scenario 2: Ceded loss = 10M × .25 = 2.5M
Ceded profit = min(20%, (2.5 × (1-20%)-2.5)/2.5) = -20%
Scenario 3: Ceded loss = 5M
Ceded profit = min(20%, (2.5 × (1-20%)-5)/2.5) = -120%
Scenario 4: Ceded loss = min(5.5,30 × .25) = 5.5M
Ceded profit = min(20%, (2.5 × (1-20%)-5.5)/2.5) = -140%

Part d: 1.5 points

Sample 1

$2{,}000{,}000 = \exp(\mu + \sigma^2/2)$
$[\ln(8{,}000{,}000) - \mu]/\sigma = 2.326$
$15.8950 - 2.326\sigma = \mu$
$2{,}000{,}000 = \exp[15.8950 - 2.326\sigma + \sigma^2/2]$
$-1.3863 = -2.326\sigma + \sigma^2/2$
$-2.7727 = -4.6520\sigma + \sigma^2$
$\sigma^2 - 4.6520\sigma + 2.7727 = 0$
$\sigma = (4.6520 \pm 3.2481)/2$
$\sigma = 0.7019$ or ~~3.9501~~
$\mu = 14.2624$ or ~~6.7071~~
$\text{stddev}(x) = \exp(0.7019^2 - 1)(\exp(2(14.2624) - 0.7019^2)^{1/2} = 1{,}595{,}790$
CV=1,595,790/2,000,000
=0.7979.

Sample 2

20% LR → E[X] = 2M
1/100 years, 8M : P(X>8m) = 0.01 or P(X<8M) = 0.99
$(\ln(x) - \mu)/\sigma = 2.326$
$E(X) = \exp(\mu + \sigma^2/2) = 2M$
$\exp(\ln(8M) - 2.326\sigma + \sigma^2/2) = 2M \rightarrow \sigma = 0.7019$
$CV(X) = \text{sqrt}(\exp(\sigma^2)-1)$
= 0.798 (since CV<1)

Part e: 1 point

Sample 1

i) $P(X>6M) = 1 - F(6M) = 1 - \Phi[(\ln(6M) - 14.3634)/0.7019]$
$= 1 - \Phi(1.916)$
$= 1 - 0.9723 = 0.0277$

ii) Negative ceded profit occurs when 2M – Loss×0.25 < 0 or Loss>8M.
We know 8M is 1 in 100 years, so prob. of negative ceded profit on option 2 is 1%.

Sample 2

i) $F(6M) = \Phi[(\ln(6M) - \mu)/\sigma] = \Phi(1.916) = .9723$ (from the table)
$P(X>6M) = 1 - F(6M) = 1 - .9723 = 2.77\%$

ii) Breakeven → Ceded losses = ceded premium & commission
25%X=10M(25%)(1-20%)
.25%X = 2M
X=8M

So ceded profit <0 if X>8M
P(X>8M) = 1/100 years = 0.01.

Part f: 0.5 point

Sample Responses for Advantage of QS

- It has a ceded loss ratio cap (220%) which limits the downside to the reinsurer
- Reinsurer can share in the profits of the primary insurer
- Insurer will have financial incentive for risk control
- Large ceded premium which can be used to earn investment income
- Easier to administer since same % is ceded on every loss
- Less volatility compared to an XS plan as the ceded amount is a fixed % from ground up
- Results should be more stable since the reinsurer assumes a fixed percentage of loss
- Learns about losses on a 1st dollar basis so reinsurer will have better knowledge of what's coming up in terms of development and will have shorter report lag

Sample Responses for Disadvantage of QS

- Large loss potential when loss experience is bad
- Reinsurer has to pay more ceding commissions
- Profit commission limits the reinsurer's upside
- Reinsurer needs to pay ceding commission up-front, may have cash flow disadvantage

Part g: 0.75 point

Sample Responses for choosing Aggregate QS

- The aggregate QS protects the insurer from adverse loss scenarios up to a 220% loss ratio while the aggregate XOL only protects a small portion ($2M) of adverse loss scenarios. The aggregate QS also has profit commission which limits the reinsurer's profit (reward insurer for good experience)
- QS has lower ceded profits and offers more coverage in bad years (when loss > $8M)
- Choose QS: more protection against tail events; profit sharing if good loss experience is realized
- Select the QS, more coverage (ground up coverage at 25%). The $2M limit from the agg XOL is small. Ceded commission seems reasonable. Profit commission in place for favorable loss experience.
- Choose QS since it will provide ceding commission, better cash flow advantage for insurer, can use the money to earn investment income

Sample Responses for choosing Aggregate XOL

- The loss ratio for the insurer's book is very stable. For a more stable and profitable LOB, an aggregate XOL is preferred so that less profit is ceded in good years and tail events can be protected with relatively lower ceded premium

- Recommend option 1 for the insurer, as it is cheaper upfront, providing a cashflow advantage and still protects against risk of potential large losses
- I would choose the aggregate excess of loss. The loss ratio is low so presumably it is profitable. The QS would be ceding profitable business whereas the Agg XOL would only cede in the event losses get too high

EXAMINER'S REPORT

Candidates were expected to test for shifting parameters for each LOB and make a projection of claim frequency for one in particular.

Candidates were then expected to analyze two reinsurance options under multiple loss scenarios for the other LOB, and evaluate both options from the perspectives of the primary insurer and the reinsurer.

Part a

Candidates were expected to understand how to test for shifting risk parameters by performing chi-squared tests on expected claim count. They were expected to test the lines separately and appropriately use earned premium as an exposure base, accounting for growth and changes in the mix of business.

To receive full credit, candidates needed to use earned premium as the exposure base, as it was the only option available.

Common mistakes included:

- Not using earned premium as an exposure base and ignoring growth when calculating expected claims
- Combining the two lines of business before testing and ignoring mix of business changes
- Selecting the wrong degrees of freedom to determine the critical value

Part b

Candidates were expected to calculate MSE for Z=0.1, determine the optimal Z value using the MSE criterion, and calculate the future frequency estimate. To do this, candidates needed to calculate what the frequency estimates would have been historically (using the cumulative frequency available up to that point in time) for Z=0.1.

The MSE values given for Z=0.5 and Z=0.9 in the question were calculated using $1000s of premium as the exposure base, but this was not identified in the question. Because candidates were not expected to recalculate these values, those that calculated frequencies in dollars of earned premium rather than in $1000s also received full credit, assuming the correct conclusions were reached using these values (see Sample 2).

Common mistakes included:

- No attempt to calculate MSE for Z=0.1 and at the same time not providing any justification for the same
- Incorrect formula for MSE
- Using the same estimated frequency for each of the prior years in the MSE calculation

- Calculating the estimated frequencies using the overall five-year frequency as the credibility complement instead of the cumulative frequency to date
- Selecting the correct Z value but neglecting to calculate a point estimate for future expected claim frequency
- Performing calculations using claims counts instead of frequencies

Part c

Candidates were expected to calculate ceded profit as defined in the question itself for each loss scenario separately. For aggregate excess of loss treaty (Option 1), the ceded profit is equal to ceded premium less ceded loss (as there was no ceding commission). For aggregate quota share treaty (Option 2), the ceded profit is equal to ceded premium less ceded losses and ceding commission. The candidate was expected to make correct adjustments for the profit commission for Scenario 1 and the aggregate ceded loss ratio cap for Scenario 4. The profit could be stated in dollars or as a percentage of ceded premium.

Some candidates averaged or aggregated the scenarios, possibly interpreting the four scenarios as individual claims or losses. Because no weights were given, this interpretation is not reasonable. However, such solutions received partial credit if candidates showed sufficient work to demonstrate their understanding of the different treaty terms used in the question.

Other common mistakes included:

- Mixing up limit and retention for excess of loss treaty, or applying rate on line to gross premium instead of the treaty limit
- Calculating target profit commission and/or loss ratio cap by applying given percentages to ceded premium net of ceding commission, or using different quota share percentages (e.g. 20% quota share)
- Mixing up ceded profit with profit commission or not recognizing negative profits

Part d

Candidates were expected to understand aggregate loss distributions with respect to the relationship between μ, σ, Var(X) and E(X), and various reinsurance structures.

Candidates were expected to understand the proper relationships in order to set up a system of equations for F(X) and E(X), and then solve for μ and σ. Candidates were then expected to calculate the coefficient of variation by either solving for Var(X) and E(X) or by recognizing that the coefficient of variation simplifies (for a lognormal random variable) to $\sqrt{e^{\sigma^2} - 1}$.

Common mistakes included:

- Using historical losses or the scenarios from part c to solve for μ and σ
- Using historical losses or the scenarios from part c to solve for Var(X) and E(X)
- Setting μ as 2,000,000
- Forgetting to take the inverse of Φ to convert 0.99 to 2.326
- Taking the inverse of Φ of 0.98 instead of 0.99
- Calculating the coefficient of variation as σ/μ

- Calculating the coefficient of variation as E(X)/sd(X)
- Using a value other than 8,000,000 as the required capital. The question specifically stated that "the Gross Capital required to support the expansion is $8,000,000"

Part e

Candidates were expected to understand the proper amount at which the aggregate excess of loss treaty attaches (that is, at the attachment point) and to understand the relationship to determine when a negative ceded profit occurs on the aggregate quota share.

Common mistakes included:

- Not recognizing that the question asked for the probability that the loss is greater than the threshold. That is, supplying F(6mil) instead of P(X>6mil) = 1 – F(6mil)
- Calculating the probability of attachment as F(8mil)-F(6mil) or $\frac{F(8mil)-F(6mil)}{1-F(6mil)}$
- Setting up the ceded profit equation incorrectly

Part f

Candidates were expected to understand types of reinsurance contracts and common provisions in reinsurance contracts. Both responses that either related directly to the specific treaties in the question, or the general characteristics of quota share reinsurance were accepted.

Common mistakes included:

- Providing advantages/disadvantages from the perspective of the primary insurer instead of the reinsurer
- Providing a response that is too vague (e.g. easy to calculate, simple to administer), unless the candidate offered appropriate justification for that answer (e.g. simpler to administer because only summary-level data is needed)
- Stating that an advantage from the reinsurer's perspective is receiving more premium without mentioning the timing of premium (cashflow benefits) or recognizing that the reinsurer is also taking on more risk
- Only providing an advantage and not providing a disadvantage (or vice versa)
- Providing two advantages or disadvantages instead of one of each (only the first provided is graded).

Part g

Candidates were expected to compare the two reinsurance options presented in the problem, make a recommendation for which option the insurer should select, and give reasonable justifications for the selection.

Common mistakes included:

- Not providing justification for the selection or providing too vague of an explanation
- Providing justifications that do not match the recommendations
- Making a recommendation from the reinsurer's perspective
- Misunderstanding aggregate excess of loss policy as per occurrence or per risk excess of loss policy
- Not understanding that the primary insurer would prefer the option with the lower ceded

profit (calculated in part c)