2. (9.5 points)

A certain golf course holds a series of tournaments throughout the year, during which prizes are awarded if a player hits a hole-in-one. The prize amounts vary based on the difficulty of the hole, but are consistent across tournaments.

The course has approached an insurer to design a policy which would protect the course from large prize payouts. The insurer would pay the full prize amount directly to the winning player, and seek recoveries of any applicable deductible from the golf course.

A single annual policy will cover all of the tournaments throughout a given year.

The total number of holes-in-one in a given year (n), across all tournaments, has the following probability distribution:

n	$P(n)$
0	0.80
1	0.15
2	0.05
3 or more	0.00

Given that a hole-in-one has occurred, the prize amount (x) has the following probability distribution:

x	$P(x)$
$100,000	0.40
$250,000	0.35
$500,000	0.24
$1,000,000	0.01

The expenses and profit for this policy are:

Fixed Expenses		None
General Expenses	8%	of premium
Commission	10%	of premium
Taxes	5%	of premium
Underwriting Profit	5%	of premium

The above assumptions apply to all program structures.

a. (1 point)

Calculate the annual guaranteed cost premium for this policy.

b. (1 point)

Calculate the annual premium for this policy under a $100,000 per-occurrence deductible.

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c. (1.75 points)

Calculate the annual premium for this policy under a $1,000,000 annual aggregate deductible and no per-occurrence deductible.

d. (1 point)

Calculate the annual premium for this policy under a $250,000 franchise deductible.

e. (1 point)

Explain which of the deductible structures above would generate the greatest credit risk to the insurer, and estimate the magnitude of that risk.

f. (1.25 points)

The insurer has allocated $45,000 of capital to support the guaranteed cost policy in part a. above.

Recommend and justify which of the three options below would be the most appropriate capital requirement to support the per-occurrence deductible policy in part b. above:

i. $45,000
ii. $32,000
iii. $19,000

g. (2.5 points)

Based on the success of this program, the insurer wishes to expand this coverage to other golf courses.

The insurer will offer varying per occurrence limits with no aggregate limits, the prize amounts would remain the same, and the same distribution assumptions would apply.

Using $100,000 as a basic limit, use the variance method to calculate the risk-loaded ILF for a $500,000 policy limit. Assume $\rho(100{,}000) = 5{,}750$ and use $k = 0.0000005$.

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QUESTION 2	
TOTAL POINT VALUE: 9.5	**LEARNING OBJECTIVE(S): B1, B2, B5, B6, B7**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1
E[N] = 0×0.8+1×0.15+2×0.05 = 0.25
E[X] = 100K×0.4+250K×0.35+500K×0.24+1M×0.01 = 257,500
E[S] = 0.25×257,500 = 64,375
Expense Multiplier = 1/(1-0.08-0.1-0.05-0.05) = 1.38889
GC Premium = 64,375×1.38889 = 89,410

Sample 2
E[N] = 0×0.8+1×0.15+2×0.05 = 0.25
E[X] = 100K×0.4+250K×0.35+500K×0.24+1M×0.01 = 257,500
E[S] = 0.25×257,500 = 64,375
GC Premium = 64,375/(1-0.08-0.1-0.05-0.05) = 89,410

Sample 3

Loss 1	Loss 2	Total Loss	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
100		100	0.15	0.4		0.06	6
250		250	0.15	0.35		0.0525	13.125
500		500	0.15	0.24		0.036	18
1,000		1,000	0.15	0.01		0.0015	1.5
100	100	200	0.05	0.4	0.4	0.008	1.6
100	250	350	0.05	0.4	0.35	0.007	2.45
100	500	600	0.05	0.4	0.24	0.0048	2.88
100	1,000	1,100	0.05	0.4	0.01	0.0002	0.22
250	100	350	0.05	0.35	0.4	0.007	2.45
250	250	500	0.05	0.35	0.35	0.006125	3.0625
250	500	750	0.05	0.35	0.24	0.0042	3.15
250	1,000	1,250	0.05	0.35	0.01	0.000175	0.21875
500	100	600	0.05	0.24	0.4	0.0048	2.88
500	250	750	0.05	0.24	0.35	0.0042	3.15
500	500	1,000	0.05	0.24	0.24	0.00288	2.88
500	1,000	1,500	0.05	0.24	0.01	0.00012	0.18
1,000	100	1,100	0.05	0.01	0.4	0.0002	0.22
1,000	250	1,250	0.05	0.01	0.35	0.000175	0.21875
1,000	500	1,500	0.05	0.01	0.24	0.00012	0.18
1,000	1,000	2,000	0.05	0.01	0.01	0.000005	0.01

The sum of the Total Loss × total prob = 64.375K
GC Premium = 64.375/(1-.08-.05-.05-.1) =89.410K

Sample 4

Loss Amount	Prob
0	0.8
100	=.15×.4 = .06
200	=.05×.4^2= .008
250	=.15×.35 = .0525
350	=.05×.4×.35×2 = .014
500	=.15×.35 + .05×.35^2 = .042125
600	=.05×.24×0.4×2 = .0096
750	=.05×.35×0.24×2=.0084
1,000	=.15×.01+.05×.24^2 = .00438
1,100	=.05×.01×.4×2 = .0004
1,250	=.05×.01×.35×2 = .00035
1,500	=.05×.01×.24×2 = .00024
2,000	=.05×.01^2 = .000005

=0×.8 + 100×.06 + 200×.008 + …. + 2,000×.000005 = 64,375
GC Premium = 64,375/(1-0.08-0.1-0.05-0.05) = 89,410

Part b: 1 point

Sample 1

x	*P(x)*
0	0.40
150,000	0.35
400,000	0.24
900,000	0.01

E[N] = 0.25
E[X] = 150K×0.35+400K×0.24+900K×0.01 = 157,500
E[S] = 0.25×157,500 = 39,375
Deductible Premium = 39,375×1.38889 = 54,688

Sample 2

Holes	Prob	Deductible
0	0.8	0
1	0.15	100,000
2	0.05	200,000

100K×.15 + 200K×.05 = 25,000
64,375-25,000 = 39,375
39,375/.72 = 54,688

Sample 3

LER = 100,000/257,500 = 0.388349
64,375×(1-.388349) = 39,375
39,375/.72 = 54,688

Sample 4

Loss 1	Loss 2	Total Loss	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
0		0	0.15	0.4		0.06	0
150		150	0.15	0.35		0.0525	7.875
400		400	0.15	0.24		0.036	14.4
900		900	0.15	0.01		0.0015	1.35
0	0	0	0.05	0.4	0.4	0.008	0
0	150	150	0.05	0.4	0.35	0.007	1.05
0	400	400	0.05	0.4	0.24	0.0048	1.92
0	900	900	0.05	0.4	0.01	0.0002	0.18
150	0	150	0.05	0.35	0.4	0.007	1.05
150	150	300	0.05	0.35	0.35	0.006125	1.8375
150	400	550	0.05	0.35	0.24	0.0042	2.31
150	900	1,050	0.05	0.35	0.01	0.000175	0.18375
400	0	400	0.05	0.24	0.4	0.0048	1.92
400	150	550	0.05	0.24	0.35	0.0042	2.31
400	400	800	0.05	0.24	0.24	0.00288	2.304
400	900	1,300	0.05	0.24	0.01	0.00012	0.156
900	0	900	0.05	0.01	0.4	0.0002	0.18
900	150	1,050	0.05	0.01	0.35	0.000175	0.18375
900	400	1,300	0.05	0.01	0.24	0.00012	0.156
900	900	1,800	0.05	0.01	0.01	0.000005	0.009

The sum of the Total Loss × total prob = 39.375K

Deductible Premium = 39.375/(1-.08-.05-.05-.1) =54.688K

Sample 5

Loss Amount	Prob
0	0.8
150	=.15×.35 + .05×.4×.35×2 = .0665
300	=.05×.35^2 = .006125
400	=.15×.24 + .05×.4×.24×2 = .0456
550	=.05×.35×0.24×2=.0084
800	=.05×.24^2 = .00288
900	=.15×.01 + .05×.01×.4×2 = .0019
1,050	=.05×.01×.35×2 = .00035
1,300	=.05×.01×.24×2 = .00024
1,800	=.05×.01^2 = .000005

=0×.8 + 150×.0665 + 300×.006125 + …. + 1,800×.000005 = 39,375

Deductible Premium = 39,375×1.3888889= 54,688

Part c: 1.75 points

Sample 1

In order to exceed the aggregate deductible, must have 2 claims and at least 1 needs to be $1M.

2nd Claim	Probability	Claim × Prob
100,000	2×.05×.01×.4 = .0004	40
250,000	2×.05×.01×.35 .00035	87.5
500,000	2×.05×.01×.24 = .00024	120
1,000,000	.05×.01×.01 = .000005	5

40 + 87.5 + 120 + 5 = 252.5

252.5/.72 = 350.7

Sample 2

Agg Loss	Prob	Agg Loss × Prob
0	0.8	0
100	.15×.4 = 0.06	6
200	.05×.4×.4 = 0.008	1.6
250	.15×.35 = 0.0525	13.125
350	2×.05×.35 = 0.014	4.9
500	.15×.24 + .05×.35×.35 = 0.042125	21.0625
600	2×.05×.4×.24 = 0.0096	5.76
750	2×.05×.35×.24 = 0.0084	6.3
1000	1-sum of above = 0.005375	5.375

6+1.6+13.125+4.9+21.0625+5.76+6.3+5.375 = 64.1225

64.375-64.1225 = .2525

.2525×1000/.72 = 350.7

Sample 3

Loss 1	Loss 2	Total Capped at $1M	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
100		100	0.15	0.4		0.06	6
250		250	0.15	0.35		0.0525	13.125
500		500	0.15	0.24		0.036	18
1,000		1,000	0.15	0.01		0.0015	1.5
100	100	200	0.05	0.4	0.4	0.008	1.6
100	250	350	0.05	0.4	0.35	0.007	2.45
100	500	600	0.05	0.4	0.24	0.0048	2.88
100	1,000	1,000	0.05	0.4	0.01	0.0002	0.2
250	100	350	0.05	0.35	0.4	0.007	2.45
250	250	500	0.05	0.35	0.35	0.006125	3.0625
250	500	750	0.05	0.35	0.24	0.0042	3.15
250	1,000	1,000	0.05	0.35	0.01	0.000175	0.175
500	100	600	0.05	0.24	0.4	0.0048	2.88
500	250	750	0.05	0.24	0.35	0.0042	3.15
500	500	1,000	0.05	0.24	0.24	0.00288	2.88
500	1,000	1,000	0.05	0.24	0.01	0.00012	0.12
1,000	100	1,000	0.05	0.01	0.4	0.0002	0.2
1,000	250	1,000	0.05	0.01	0.35	0.000175	0.175
1,000	500	1,000	0.05	0.01	0.24	0.00012	0.12
1,000	1,000	1,000	0.05	0.01	0.01	0.000005	0.005

Sum of Total Loss × Total Prob = 64.1225
64.375-64.1225 = .2525
.2525×1000/.72 = 350.7

Part d: 1 point

Sample 1

Eliminate losses 250K and under, pay full amount for claims greater than $250K
500K×.4 + 1M×.01 = 130K
130K×.25 = 32,500
32,500/.72 = 45,139

Sample 2

E[X:250K] = 100K×.4 + 250K×.35 + 250K×.24 + 250K×.01 = 190,000
F(250,000) = .75
LER = (190,000 – 250,000×(1-.75)) / 257,500 = .4951
89,410×(1-.4951) = 45,139

Part e: 1 point

Sample 1

The $1M aggregate deductible generates the most credit risk as the insurer will make all payouts and then collect from the insured payouts at or below the deductible.

The magnitude of the credit risk is the amount of expected losses below the deductible, which equals \$64,375 – \$252.5 = \$64,122.5.

Sample 2
The insurer pays the prize amount immediately and seeks reimbursement from the insured.

Under the deductible options, the maximum credit risk is:

- 100K Deductible: 2 × 100,000 = \$200,000 (2 hole-in-ones)
- 1M Aggregate: \$1,000,000 (1 or 2 hole-in-ones totaling at least \$1M)
- 250K Franchise Deductible: 2 × 250,000 = \$500,000 (2 hole-in-ones of \$250,000)

Therefore, the \$1M aggregate deductible generates the most credit risk.

Part f: 1.25 points

Sample 1
The needed capital under the deductible will be less than that needed under the GC policy, so eliminate the \$45k option.

From part a, E(S) = 64,375
From part b, E(S) = 39,375

45,000 × 39,375/64375 = 27,524

The deductible covers excess losses which are more volatile, so the capital required should be greater than \$27.5k. Therefore \$19k is too low; select \$32k.

Sample 2
$Var(S) = E(N) \times Var(X) + Var(N) \times E(X)^2$

Under the GC policy, Var(S) = 2.6455×10^{10}
Under the deductible option, Var(S) = 1.4524×10^{10}

The ratio of the standard deviations is then 1.35
45000/1.35 = 33,333 so select a capital level of \$32k

Sample 3
Return on Capital = Profit/Capital

Under the GC Policy, ROC = 5% × 89410 / 45000 = 9.93%

To keep this same return of 9.93% based on premium of 54688 from part b, the needed capital is 27,520. Accounting for the addition of credit risk, select the \$32k option.

Part g: 2.5 points

Sample 1

$\rho(l) = k[E(X^2:l) + \delta \times E(X:l)^2]$

$E[X:100,000] = 100,000$

$E[X:100,000^2] = 100,000^2$

$5750 = 0.0000005(100000^2 + \delta \times 100000^2) \rightarrow \delta = 0.15$

$E[X: 500,000] = .4 \times 100,000 + .35 \times 250,000 + .25 \times 500,000 = 252,500$

$E[X^2:500,000] = .4 \times 100,000^2 + .35 \times 250,000^2 + .25 \times 500,000^2 = 8.8375 \times 10^{10}$

$\rho(500,000) = 0.0000005 \times (8.8375 \times 10^{10} + 0.15 \times 252,500^2) = 48,969$

ILF = [252,500 + 48,969)/(100,000+5,750) = **2.851**

Sample 2

$\rho(l) = k[E(X^2:l) + \delta \times E(X:l)^2]$

$\delta = Var(N)/E(N) - 1$

$E(N) = 1 \times .15 + 2 \times .05 = .25$

$Var(N) = (0^2) \times .8 + (1^2) \times (.15) + (2^2) \times .05 - (0.25^2) = .2875$

$\delta = .2875/.25 - 1 = .15$

$\rho(500K) = k[E(X^2:500K) + \delta E(X:500K)^2]$

$E(X^2:500K) = 100K^2 \times .4 + \ldots\ldots 500K^2 \times (.24+.01) = 8.8375 \times 10^{10}$

$\rho(500K) = k[8.8375 \times 10^{10} + 0.15 \times 252,500^2] = 48,969$

$I(500K) = [E(X:500K) + \rho(500K)]/[E(X:100K) + \rho(100K)] = (252,500 + 48,969)/(100,000 + 5,750)$

I(500K) = **2.851**

Sample 3

$ILF = [E(X:500K) + \rho(500K)]/[E(X:100K) + \rho(100K)]$

Assuming frequency doesn't change by limit, so only looking @severity.

E(X:100K) = 100,000 (all losses are above 100K or equal)

$E(X:500K) = .4(100K) + .35(250K) + .25(500K) = 252,500$

$\rho(100K) = 5,750$

Assume expenses don't vary by limit

$V(N) = E(N^2) - E(N)^2 = [.15(1^2) + .05(2^2)] - [.25]^2 = .2875$

$V(S) = E(N)V(X) + \{[E(X)]^2 \times V(N)\} = (.25) \times [(.4)(100K^2)+(.35)(250K^2)+(.25)(500K^2) - (252,500)^2] + (252,500^2)(.2875) = 2.448 \times 10^{10}$

$\rho(500K) = .0000005[V(S)/E(N)] = 48,969$

ILF = (252,500 + 48,969) / (100,000 + 5,750) = **2.85**

EXAMINER'S REPORT

Candidates were expected to be able to calculate annual premium with several different features, including guaranteed cost premium, premium with a per-occurrence deductible, premium with an aggregate deductible, and premium with a franchise deductible. Candidates were then expected to quantify and discuss the relative credit risk of these deductible options and compare capital allocated to guaranteed cost vs. per occurrence deductible policies. Finally, candidates were expected to calculate a risk-adjusted ILF at a given limit.

Part a

Candidates were expected to understand the concepts of frequency, severity and expense multipliers to calculate guaranteed cost premium.

Several candidates developed a discrete distribution of aggregate losses, mapping out each possible permutation. While this is a valid approach, it would have been very time-consuming and would have unnecessarily complicated the solution. Candidates who took this approach had notably more calculation errors in their solutions.

Common mistakes included:

- When approaching the question using the full aggregate distribution: ,
    - Missing a combination of losses when n=2
    - Forgetting to multiply some loss combinations by 2. For example, if Loss 1 = 250 and Loss 2 = 500, the probability of that combination ought to be multiplied by 2 because you could also have Loss 1 = 500 and Loss 2 = 250.
- Using the wrong frequency (used 0.2 instead of 0.25)
- Incorrectly treating part of the expenses as fixed

Part b

Candidates were expected to demonstrate how a per-occurrence deductible works, correctly calculate a new severity, apply the correct frequency and correct expense multiplier to determine the deductible premium.

Common mistakes included:

- When approaching the question using the full aggregate distribution:
    - Missing a combination of losses when n=2
    - Forgetting to multiply some loss combinations by 2. For example, if Loss 1 = 250 and Loss 2 = 500, the probability of that combination ought to be multiplied by 2 because you could also have Loss 1 = 500 and Loss 2 = 250.
- Calculating the premium for the loss under the deductible instead of the excess layer

Part c

Candidates were expected to recognize that in order to reach the aggregate deductible, there must be two claims and at least one of them has to be $1M. The candidate needed to determine the correct probability distribution, apply it to the losses and apply the expense multiplier to arrive at the correct annual premium.

Full credit was not awarded if the candidate treated the $1M as an aggregate limit rather than an aggregate deductible (the problem stated that $1M was the aggregate deductible).
Credit was not awarded to candidates who treated the deductible as a per-occurrence deductible.

Common mistakes included:

- Not including all possible loss combinations (i.e. only including possible aggregates of $2M and $1.5M, but omitting the $1.1M and $1.25M aggregate combinations)
- For candidates who calculated the aggregate loss distribution, missing some combinations due to the volume of possibilities

- Not multiplying the aggregate losses of $1.1M, $1.25M and $1.5M by 2
- Not multiplying by the correct frequency
- Treating the problem like an aggregate limit rather than an aggregate deductible
- Not subtracting the aggregate deductible of $1M from the loss distribution

Part d

Candidates were expected to demonstrate how a franchise deductible works by applying the correct loss distribution to determine severity, multiply by the correct frequency and expense multiplier to arrive at the correct annual premium. Full credit was not given if a $250K loss was treated as if there were no deductible and paid on a ground up basis.

Common mistakes included:

- Not including all possible loss combinations
- Not multiplying loss combinations by 2 where Loss 1 does not equal Loss 2
- Assuming that losses of $250K should be paid in full (from Bahnemann, pg. 178: "The franchise deductible eliminates all claims less than or equal to the deductible or 'franchise' amount *d*, and claims in excess of *d* are paid in full.")
- Treating the franchise deductible as an aggregate deductible rather than a per occurrence deductible

Part e

Candidates were expected to consider the credit risk of the three deductible options presented and opine on the option with the largest credit risk. Additionally, candidates were expected to quantify the amount of credit risk.

Most candidates were successful in identifying that credit risk arises from the insurer seeking reimbursement from the insured for coverage of payouts below the deductible. Considering the expected losses below the various deductible options or the maximum amount of reimbursable losses were both acceptable ways to quantify the magnitude of the credit risk.

Common mistakes included:

- Incorrectly interpreting the coverage structure of the deductible options
- Failing to quantify the magnitude of the credit risk

Part f

Candidates were expected to select and justify, from the list of given options, a capital amount the insurer could hold to support the $100k per-occurrence deductible.

Almost all candidates recognized that the deductible reduces the needed capital as compared to the guaranteed cost policy.

Candidates took a variety of approaches to quantify the reduction in capital. Comparing the reduction in premium or the reduction in losses from the application of the deductible were most common. Other acceptable responses included relating the reduction in capital to the standard deviations of the two policies, using a PML-based approach to justify the capital selection, or contemplating a return on capital to support the selection.

Finally, to receive full credit candidates were expected to recognize that because the insurer is providing coverage for the more volatile excess layer, the reduction in capital is less than the amount of losses eliminated by the application of the deductible. Stating the introduction of credit risk reduces the overall reduction in capital was also accepted.

Common mistakes included:

- Stating that the capital required should be equal to the expected losses
- Not recognizing that the volatility of excess losses and/or the addition of credit risk causes the reduction in capital to be less than the reduction of primary losses

Part g

Candidates were expected to calculate the risk-loaded ILF at the given increased limit, which involved calculating delta, limited expected severities, and risk loads.

Common mistakes included:

- Using limited loss (frequency × severity) instead of just limited severity
- Using an incorrect formula for the risk load (such as not multiplying the inner terms by k or not squaring E(X:l) in the second term)
- Calculating E(X:l), $E(X^2:l)$ or ρ(l) incorrectly