13. (2.75 points)

An insurance company observes the following ground-up claim experience for a book of business:

- Claim counts (N) follow a Poisson distribution with $\lambda = 5{,}000$
- Claim size (X) follows a Pareto distribution with $\alpha = 2$ and $\beta = 6{,}000$

The insurance company pays claims in excess of $10,000.

Given the following:

- For a Poisson distribution:

$$\Pr(N = n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

- For a Pareto distribution:

$$f(x) = \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^\alpha$$

$$E[X;x] = \frac{\beta}{\alpha-1}\left[1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]$$

a. (0.75 point)

Calculate the expected number of claims in excess of $10,000.

b. (2 points)

Assume claim size is subject to a uniform annual inflation rate of 3%.

Calculate the rate at which ground-up claim counts must change in order for there to be no change in expected annual total aggregate excess losses.

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QUESTION 13	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): B1, B2**
SAMPLE ANSWERS	
Part a: 0.75 point	

Sample 1

P(X>10,000) = 1 – F(10,000)

$= \left(\frac{\beta}{\beta+10{,}000}\right)^{\alpha} = \left(\frac{6{,}000}{6{,}000+10{,}000}\right)^{2}$ = 0.140625

E[N >10,000] = E[N] × P(X>10,000) = 5,000 (0.140625) = 703.125

Sample 2

Since Poisson, can use E[N] × [1- F(10,000)]

E[N] = λ = 5,000

F(10,000) = 1 – $\left(\frac{6{,}000}{16{,}000}\right)^{2}$ = 55/64

1- F(10,000) = 9/64

=> Expect 9/54 × 5,000 = 703.125 excess claims

Part b: 2 points

Sample 1

Change in agg losses:

$\frac{1.03\times\left(E[X]-E[X;{}^{10000}/_{1.03}]\right)}{E[X]-E[X;10000]} = \frac{1.03\times(6000-3708)}{6000-3750}$ = 1.0491

E[X] = $\frac{\beta}{\alpha-1}$ = 6000

E[X; 10000] = $\frac{6000}{2-1}\times\left(1-\left(\frac{6000}{16000}\right)^{2-1}\right)$ = 3750

$E[X;{}^{10000}/_{1.03}]$ = 3708

For $\tilde{\tau}_S$ = 1.00 (no change in agg), must have claim count impact

$\tilde{\tau}_N$ = 1 / 1.0491 = 0.9532

So % change in ground up counts = 0.9532 – 1 = -4.68%

Sample 2

E[S] = E[N] × $(E[X]-E[X;10{,}000])$ α=2 ; $\beta = 6{,}000$

= 5000 × $\left(\frac{\beta}{\alpha-1}-\frac{\beta}{\alpha-1}\left[1-\left(\frac{\beta}{10000+\beta}\right)^{\alpha-1}\right]\right)$

= 5000 × $\left(\frac{6000}{2-1}-\frac{6000}{2-1}\left[1-\left(\frac{6000}{16000}\right)^{2-1}\right]\right)$

= 50000 × 60000 × 6/16 = 11,250,000

E[S'] = E[N'] × $(E[Y]-E[Y;10{,}000])$ Y = 1.03X

= E[N'] × 1.03 × $(E[X]-E[X;10{,}000/1.03])$

= 1.03 × E[N'] × $\left(6000-6000\left[1-\frac{6000}{60000+9708}\right]\right)$

= 1.03 × 6000 × $\frac{6000}{15708}$ × E[N'] = 11,250,000

E[N'] = 4766 => rate = $\frac{4766-5000}{5000}$ = -4.68%

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to know how to calculate the expected number of excess claims, how to calculate expected loss, how to calculate excess and/or limited loss, and how inflation impacts loss calculations. The candidate was then expected to know how to derive the aggregate impact of inflation on excess loss and how this is impacted by a change in ground-up claim costs.
Part a
The candidate was expected to know how to use the parameters and formulas given on the Poisson claim count distribution and Pareto claim size distribution in order to calculate the expected number of claims in the excess layer. Common mistakes included: • Calculating the number of claims not hitting the excess layer • Calculating the probability of an excess claim, but stopping before calculating the number of claims in the excess layer • Using $\alpha - 1$ in the probability formula instead of α • Using F(x) instead of 1-F(x) to get the expected number of claims in the excess layer
Part b
Candidates were expected to know how to take claim count and claim size distributions and use them to determine the rate of change necessary in ground-up claims counts to offset the impact of inflation in a specified layer of aggregate excess losses. Most successful candidates took one of two paths to calculate the rate of change needed. The first set of candidates calculated the expected loss, limited expected loss, and modified expected loss need in the $\tilde{\tau}_S$ calculation, and then calculated the rate of change needed as $1/\tilde{\tau}_S - 1$. The second set of candidates calculated the expected total aggregate excess loss before inflation and then calculated the expected ground-up claims needed in order for that aggregate figure to stay the same after inflation. The candidates then derived the rate of change as the new ground-up claims divided by the old ground-up claims minus one. Common mistakes included: • Calculating the aggregate change as 1.0491 and saying the rate which ground-up claims needed to change was (1.0491-1) = 4.91% • Transposing the numerator and denominator in the excess loss ratio comparison • Using limited loss in the ratio comparison instead of excess loss • Stating a new potential ground-up claim number and not the rate ground-up claims needed to change • Misapplying the impact of inflation on excess loss occurrence probability in the calculation, when trying to calculate the total change to the aggregate excess layer