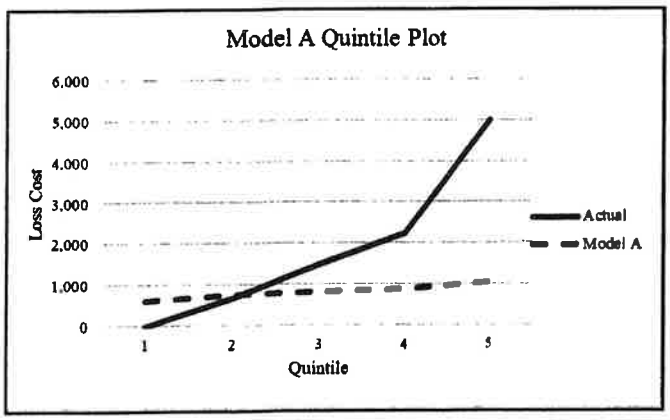
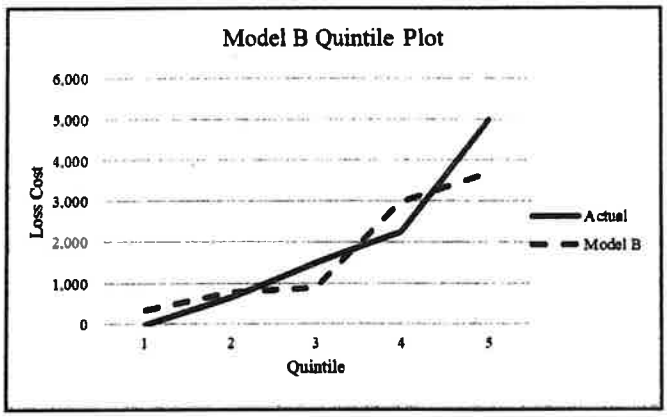
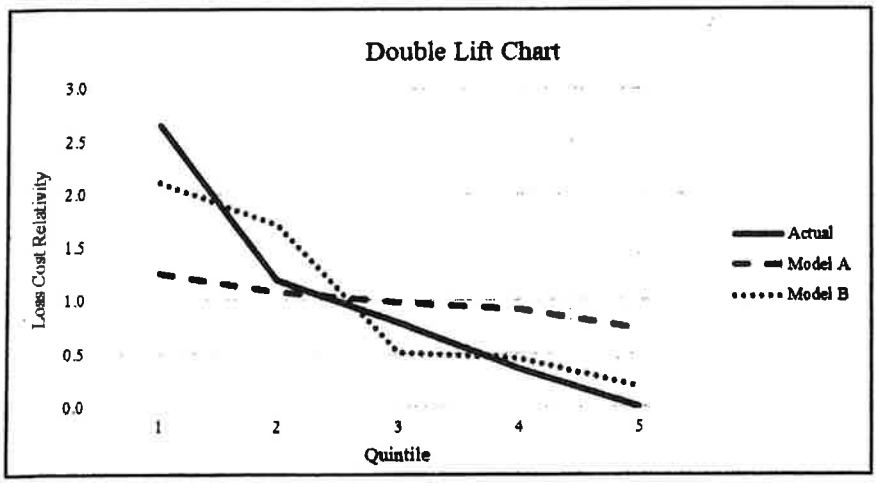
2. (2.75 points)

An actuary has built two generalized linear models to predict loss costs. Management has requested a series of model validation plots to demonstrate the appropriateness of each of the new models. Output for each model, simple quintile plots, and a double lift plot are shown below:

Observation	Actual Loss Cost	Model A Loss Cost	Model B Loss Cost	Earned Premium
1	1,500	825	900	1,800
2	675	765	800	1,450
3	0	615	350	2,375
4	2,250	900	3,000	2,625
5	5,000	1,050	3,700	4,875







Given the following:

- The actuary has already provided management with the simple quintile plots and the double lift chart shown above.
- The company has implemented several segmented rate changes over the last three years.

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a. (1 point)

For each model, provide a loss ratio plot that management can use to assess lift. Identify the basis of sorting the data.

b. (0.75 point)

Briefly describe one drawback of each type of model validation plot that the actuary has provided to management, including the plot produced in part a. above.

c. (1 point)

Using all three types of model validation plots provided to management, recommend which model should be implemented. Do not perform any calculations.

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QUESTION 2	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A.4**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

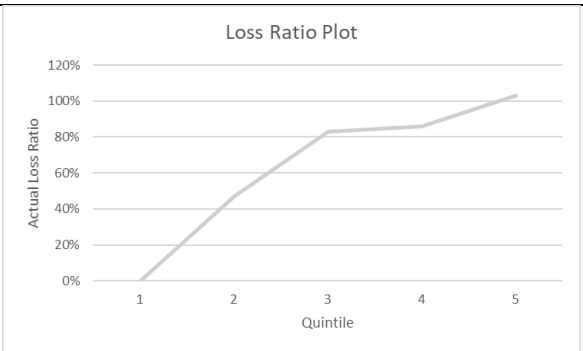
Observation	Actual LR
3	0
2	0.47
1	0.83
4	0.86
5	1.03

The data is sorted by modeled loss cost, ascending. The loss ratio plot will be the same for both models as they rank/put the observations in the same order.

Sample 2

Observation	Actual LR
3	0%
2	47%
1	83%
4	86%
5	103%

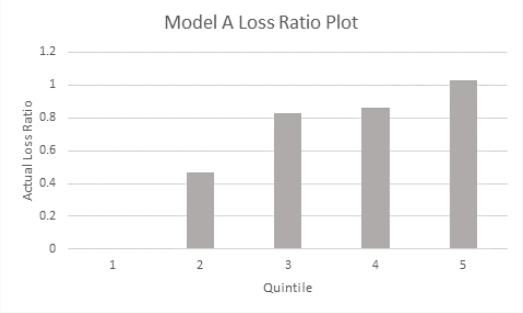
Same sort order so plot is the same. Sorted by model A and B predicted loss costs.



Sample 3

Observation	Actual LR
3	0
2	0.47
1	0.83
4	0.86
5	1.03

Sort by the model prediction (loss cost here) in ascending order. Models A and B have the same sort.



Sample 4

Sort by model predicted loss ratio for model A and B.

For Model A:

Observation	Predict LR	Actual LR
5	.22	1.03
3	.26	0
4	.34	.86
1	.46	.83
2	.53	.47

For Model B:

Observation	Predict LR	Actual LR
3	.15	0
1	.50	.83
2	.55	.47
5	.76	1.03
4	1.14	.86

Part b: 0.75 point

Sample 1

- The two LR charts look the same because when sorted, Model A & B have the same ordering
- The double lift chart is difficult to explain to management
- We need to produce two quintile plots to compare A & B, so it is more work

Sample 2

- LR Chart – only can tell how well each model identifies differences in risks, not if the predictions are accurate
- Quantile Plot – graphs are on separate charts and can only compare by looking at 2 charts
- Double Lift – compares where model A disagrees with model B most (since sorted based on this ratio) so can be harder to interpret

Sample 3

- Quintile: Need to sort groups into quintiles with approximately equal exposures which could be difficult. If there are a few large risks and many small risks, results could be skewed
- Double Lift: the sort order of model A / model B is unintuitive
- Loss Ratio: only assesses how well the GLM differentiates risks; no validation of predicted losses

Sample 4

- Quintile plot is less of a direct model comparison because it requires separate plots for each model
- Double lift charts are harder for business partners to interpret
- The LR data (EP) must be on-leveled and it may not have been here

Sample 5

- Quintile Plots: Model A output much lower loss costs, which likely wouldn't be implemented. This makes for an unfair comparison to model B
- Double Lift Chart: This plot normalizes everything and ignores that model A output lower loss costs in aggregate
- Loss Ratio Plot: This shows how well the model does at identifying risk but not at model performance

Sample 6

- Quintile Plot: Does not normalize predictions, which can make it difficult to compare one model to another
- Double Lift Plot: Does not provide information about actual loss dollars
- Loss Ratio Plot: Does not provide actual model predictions; only the order. Also not clear what basis earned premium is on. If it's not on-level, this chart can be misleading.

Part c: 1 point

Sample 1

- Based on simple quintile plot, model B is much better at predicting actual loss cost (two lines are closer)
- Based on double lift chart, model B line is also better at predicting the actual loss cost (model B & actual line are closer than model A & actual)
- Based on loss ratio chart, both models perform equally at segmenting good and bad risks
- I recommend Model B

Sample 2

- Single Quintile Plot: Model B has a better match for the model loss cost and actual loss cost
- Double Lift Chart: Model B has a better match for the model B loss cost and actual loss cost
- Loss Ratio Plot: There is upward trend in model B plot indicating it outperforms the current rating plan
- I would recommend management implement model B

Sample 3

- The loss ratio plots for both models are identical but do indicate the models successfully recognize the differences between the risks
- The quintile and double lift charts show that while model A is monotonically increasing, the model B has greater predictive accuracy
- I would recommend implementing model B

EXAMINER'S REPORT

Candidates were expected to create and assess validation plots that can be used to compare models. Candidates were also expected to understand shortcomings of model validation plots.

Part a

Candidates were expected to create loss ratio plots for Model A and Model B. The syllabus reading states that observations should be sorted by model prediction (which this question stated was loss cost) when creating loss ratio plots. Sorting by predicted loss ratio was an acceptable full credit alternative but required additional work and is not necessarily superior to sorting by predicted loss cost.

Common mistakes included:

- Sorting observations by something other than predicted loss cost or predicted loss ratio
- Plotting predicted loss ratio instead of actual loss ratio
- Calculation errors when determining the actual and/or predicted loss ratios
- Mislabeling or forgetting to label plots.

Part b

Candidates were expected to explain a drawback of simple quintile plots, double lift plots, and loss ratio plots.

Common mistakes included:

- Providing drawbacks of model validation plots in general rather than for one of the specific plots
- Discussing what is shown on each plot without providing an explanation of an actual drawback or shortcoming
- Discussing drawbacks of implementing the Model A or Model B instead of more general drawbacks of the model validation plots.

Part c

Candidates were expected to assess Model A and Model B using the simple quintile plots, double lift chart, and the loss ratio charts produced in part a. Candidates were also expected to use this assessment to recommend a particular model.

Common mistakes included:

- Stating that a plot showed that a model had more lift without any additional explanation
- Using only one or two of the model validation plots instead of all three types
- Comparing the vertical distance between the first and last quintiles of the modeled loss costs instead of the actual loss costs in the simple quintile plots
- Comparing actual loss ratios to modeled loss ratios in the loss ratio plot

- Discussing the relative performance of each model without providing a final recommendation.

QUESTION 3	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): A1**
SAMPLE ANSWERS	
Part a: 1.25 points	

Sample 1

z = 0.02 r = 10

$$.101 = \frac{10p}{1-p}$$
$$.101 - .101p = 10p$$
$$p = .01$$
$$\Pr(N = 0) = \binom{9}{0}(1 - .01)^{10}(.01)^{0}$$
$$\Pr(N = 0) = .9044$$
$$R = \frac{1}{1-\Pr(N=0)}$$
$$R = \frac{1}{1-.9044} = 10.458$$

$$Mod = (.02)(10.458) + (1 - .02)$$
$$Mod = 1.1892$$

Sample 2

$$E(x) = \frac{pr}{1-p}$$
$$.101 = \frac{p * 10}{1-p}$$
$$1 - p = 99p$$
$$p = .01$$
$$R = \frac{1}{1-(1-.01)^{10}}$$
$$R = 10.458$$
$$Mod = Z * R + (1 - Z)$$
$$Mod = (.02)(10.458) + (1 - .02)$$
$$Mod = 1.18916$$

Sample 3

$$\frac{pr}{1-p} = 0.101 = \frac{10p}{1-p}$$
$$0.101 - 0.101p = 10p$$
$$p = .009999$$

$$(1-p)^{10} = (1 - .009999)^{10} = 0.904391119$$