6. (2 points)

An actuary creates a generalized linear model (GLM) to estimate commercial property claim frequency by occupancy class and amount of insurance (AOI) for sprinklered and non-sprinklered risks. Given the following:

- Occupancy class is a categorical variable with four levels: class 1, 2, 3 and 4.
- Sprinklered status is a categorical variable with two levels: sprinklered and non-sprinklered.
- The natural log of AOI, ln(AOI), is a continuous variable.
- The log link function is selected.
- An interaction variable is included as ln(AOI) for sprinklered and zero otherwise.
- The model results are as follows:

| **Parameter** | **Coefficient** |
|---|---|
| Intercept | -8.4607 |
| Occupancy class 2 | 0.2714 |
| Occupancy class 3 | 0.3620 |
| Occupancy class 4 | 0.0395 |
| Sprinklered | 0.7228 |
| ln(AOI) | 0.4311 |
| Sprinklered:Yes, ln(AOI) | -0.0960 |

a. (0.75 point)

Calculate the ratio of the estimated model frequency of a sprinklered property to that of a non-sprinklered property for AOI = 200,000 and occupancy class 2.

b. (0.75 point)

Calculate the intercept term if AOI is centered at the base level of 200,000.

c. (0.5 point)

Briefly describe two advantages of centering variables of a GLM at their base levels.

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**SAMPLE ANSWERS AND EXAMINER'S REPORT**

| **QUESTION 6** | |
|---|---|
| **TOTAL POINT VALUE: 2** | **LEARNING OBJECTIVE(S): A3** |
| **SAMPLE ANSWERS** | |
| **Part a:** 0.75 point | |

*Sample 1*

Sprinklered Property, AOI 200K, Class 2 Fitted Frequency

$= e^{-8.4607+0.2714+\ 0.7228+\ 0.4311\ln(200000)-0.0960(1)\ln(200000)}$

$= 0.034176$

Non Sprinklered Property, AOI 200K, Class 2 Fitted Frequency

$= e^{-8.4607+0.2714+0.4311*\ln(200000)}$

$= 0.053543$

Ratio = 0. $034176$/0.053543 = 0.638286

*Sample 2*

μ for sprinklered /μ for non-sprinklered

$= e^{.7228\ -.096\ln(200000)}$ (other terms cancel out)

= 0.6383

**Part b: 0.75** point

*Sample 1*

New Intercept = -8.4607 + 0.4311ln(200,000) = -3.1987

*Sample 2*

Centered at base level of 200,000

➔ For 200,000 AOI, no coefficient

For non-sprinklered:

-8.4607 + 0.2714 + 0.4311ln(200,000) = -2.927 = b0 + 0.2714

New intercept = -3.1987

**Part c:** 0.5 point

*Transformed intercept being indicated predicted target at base case*

- Intercept represents all variable at their base levels -> easier to interpret
- The intercept term reflects average frequency at base levels, which is intuitive
- The intercept term represents your (untransformed) base rate

*Sign of interacted variable*

- Avoids non-intuitive interaction terms, such as a negative coefficient for low AOI non-sprinklered properties when base is not the center

- When variable is not centered, sometimes a coefficient may have the opposite sign than expected, this is especially true when an interaction term is present, so the coefficients are more intuitive to understand when centering variables
- When terms are not centered, you can have unintuitive results. E.g. the sprinkler coefficient is positive which can appear to indicate a higher frequency for sprinklered building

**EXAMINER'S REPORT**

Candidates were expected to know the components of a GLM formula and be able to calculate the output of the model based on the information provided. Also, they were expected to understand the transformation of variables and its impact on GLM output.

**Part a**

Candidates were expected be able to calculate the predicted frequency based on the output of the GLM model.

Common mistakes included:

- Providing the calculation for only one of Non-Sprinklered or Sprinklered
- Failing to recognize the log link function
- Error on the interaction term while calculating the frequency for Sprinklered.

**Part b**

Candidates were expected to understand how transformation of a continuous variable (Centering) and its impact on GLM output.

Common mistakes included:

- Not recognizing the new intercept as function of coefficient for ln(AOI)
- Including the interaction term in the adjustment
- Not applying additive adjustment to the original intercept.

**Part c**

Candidates were expected to provide advantages of transforming (centering) continuous variables while building a GLM.

Common mistakes included:

- Simply stating "easy to explain", "intuitive", "easy to calculate" without further detail
- Stating "p-value reduction, narrower confidence intervals, standard error reduction, and increased variable significance due to large exposure concentration" without explicit specification of the variable (Note that coefficients and their significance around "ln(AOI)" and "ln(AOI) and sprinklered" interaction do not change after the centering of AOI– refer to the GLM output on pages 56 and 57 of Goldburd et al. Credit was provided if the candidate has explicitly specified this rationale around the coefficient of "Sprinklered".)

- Stating the reduction of a variable in the model or degree of freedom as an advantage – transforming a continuous variable does not reduce the number of variables used in the GLM model, therefore, does not reduce the degree of freedom.

**QUESTION 7**

**TOTAL POINT VALUE: 7.5** | **LEARNING OBJECTIVE(S): B1-B3, B6-B7, C3**

**SAMPLE ANSWERS**

**Part a:** 2.25 points

*Sample 1*

E[X] = $\frac{\beta}{\alpha-1}$

E[X;x] = $\frac{\beta}{\alpha-1}$ [1-$\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}$]

LER = $\frac{E[X;100k]}{E[X]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3} = 0.3966$

Expected limited loss 2016-18

= 1,064,000 x 0.3966 $\left(\frac{1}{1.286*1.045^2}+\frac{1}{1.094*1.045^3}+\frac{1}{1.052*1.045^4}\right)$

= 974,860

*Sample 2*

Limited Loss % = $\frac{E[X;L]}{E[X]}$ = $\frac{\frac{\beta}{\alpha-1}}{\frac{\beta}{\alpha-1}\left[1-\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3} = 0.3965$

Annual limited expected loss = 1064 (.3965) = 421.9

-use 3 years of data lagged 1 year (use 16, 17, 18)
-average accident date is 7/1/2020, for 2018 this is 2 years of trend

| | A | B | C | AxBxC |
|---|---|---|---|---|
| PY | Prospective Lim Loss | Detrend | De-develop | E[Lim Loss at Reported] |
| 18 | 421.9 | $1.045^{-2}$ | 1/1.286 | 300.4 |
| 17 | 421.9 | $1.045^{-3}$ | 1/1.094 | 337.9 |
| 16 | 421.9 | $1.045^{-4}$ | 1/1.052 | 336.3 |
| | | | | ∑ = 974.6k |