7. (7.5 points)

An actuary is pricing a workers compensation policy. Given the following:

Policy Effective Date	January 1, 2020
Policy Term	One year
Annual Loss Trend	4.5%
Cap for individual claims	$100,000
Credibility factor	0.40
Expected ultimate loss before modification	$1,064,000

As of June 30, 2019, ground up reported losses are:

Policy Year	Total Reported Loss	Individual Claims over $100,000	
2016	$392,457	$128,305	
2017	$1,013,863	$525,626	$152,860
2018	$459,798	$275,865	
2019	$181,325	None	

The following limited loss development factors (LDFs) apply to this policy:

Maturity to Ultimate	Limited LDF
42 months	1.052
30 months	1.094
18 months	1.286

Based on an internal study, the actuary believes that claim severity follows a shifted Pareto distribution with $\alpha = 1.3$ and $\beta = 22{,}800$.

CONTINUED ON NEXT PAGE

The following expenses apply to this policy:

Loss Adjustment Expense (LAE)	7.5% of loss
Taxes and Fees	3.5% of gross premium
Acquisition	17% of gross premium
Profit and Contingencies	0% of gross premium

a. (2.25 points)

Calculate the expected ground up reported loss limited to $100,000 per claim for this insured for the three years of experience combined.

b. (1.5 points)

Calculate the total modified ground up unlimited expected loss for this policy.

c. (0.25 point)

Alternatively, the actuary could have trended and developed reported losses to the cost level of the prospective policy period. Briefly explain why this approach would not produce an identical modification factor.

d. (0.5 point)

Calculate the guaranteed cost premium for this insured.

CONTINUED ON NEXT PAGE

The insurer's management is concerned about capacity on risks of this type. To address these concerns, they are requiring facultative reinsurance to support this account. Under the treaty, the reinsurer would assume all aggregate losses between $2,000,000 and $4,000,000. The primary insurer will retain all LAE.

The primary insurer's actuary believes the following Table M and the associated expected loss groupings (ELGs) are appropriate for risks of this type:

ELG	Loss Range
31	$730,000 – 820,000
30	$820,001 – 930,000
29	$930,001 – 1,090,000
28	$1,090,001 – 1,280,000
27	$1,280,001 – 1,515,000
26	$1,515,001 – 1,844,000

	ELG					
Entry Ratio	31	30	29	28	27	26
1.50	0.1876	0.1764	0.1649	0.1529	0.1442	0.1343
1.75	0.1490	0.1376	0.1257	0.1131	0.1048	0.0963
2.00	0.1195	0.1083	0.0964	0.0838	0.0762	0.0692
2.25	0.0968	0.0859	0.0745	0.0623	0.0555	0.0497
2.50	0.0791	0.0687	0.0579	0.0465	0.0404	0.0357
2.75	0.0652	0.0554	0.0453	0.0347	0.0295	0.0257
3.00	0.0541	0.0450	0.0356	0.0260	0.0215	0.0185
3.25	0.0452	0.0367	0.0282	0.0196	0.0157	0.0133
3.50	0.0380	0.0302	0.0224	0.0148	0.0115	0.0096
3.75	0.0321	0.0250	0.0179	0.0111	0.0084	0.0069
4.00	0.0273	0.0207	0.0144	0.0084	0.0062	0.0050

e. (1.5 points)

Calculate the loss expected to be ceded to the reinsurer under this treaty. Round to the nearest entry ratio (i.e., do not interpolate).

f. (1 point)

A reinsurer has quoted a premium of $200,000 for this treaty. Calculate the premium the primary insurer must charge in order to maintain the same underwriting profit.

g. (0.5 point)

Explain how the assumptions of the primary insurer's actuary may have resulted in an inequitable premium calculated in part f. above.

CONTINUED ON NEXT PAGE

- Stating the reduction of a variable in the model or degree of freedom as an advantage – transforming a continuous variable does not reduce the number of variables used in the GLM model, therefore, does not reduce the degree of freedom.

QUESTION 7

TOTAL POINT VALUE: 7.5 | **LEARNING OBJECTIVE(S): B1-B3, B6-B7, C3**

SAMPLE ANSWERS

Part a: 2.25 points

Sample 1

E[X] = $\frac{\beta}{\alpha-1}$

E[X;x] = $\frac{\beta}{\alpha-1}$ [1-$\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}$]

LER = $\frac{E[X;100k]}{E[X]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3} = 0.3966$

Expected limited loss 2016-18

= 1,064,000 x 0.3966 $\left(\frac{1}{1.286*1.045^2}+\frac{1}{1.094*1.045^3}+\frac{1}{1.052*1.045^4}\right)$

= 974,860

Sample 2

Limited Loss % = $\frac{E[X;L]}{E[X]}$ = $\frac{\frac{\beta}{\alpha-1}}{\frac{\beta}{\alpha-1}\left[1-\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3} = 0.3965$

Annual limited expected loss = 1064 (.3965) = 421.9

-use 3 years of data lagged 1 year (use 16, 17, 18)
-average accident date is 7/1/2020, for 2018 this is 2 years of trend

	A	B	C	AxBxC
PY	Prospective Lim Loss	Detrend	De-develop	E[Lim Loss at Reported]
18	421.9	1.045^{-2}	1/1.286	300.4
17	421.9	1.045^{-3}	1/1.094	337.9
16	421.9	1.045^{-4}	1/1.052	336.3
				∑ = 974.6k

Alternate Solution:
Experience period: PY 2016-2018
*Assume "E(loss) before mod" refers to limited losses before experience mod

PY	E(Ult loss)	
2016	848,127	detrend 7/1/18-7/1/2020
17	852,267	undeveloping loss
18	757,649	= $1{,}064{,}000/(1.045)^2/1.286$
Total	2,458,042	

Part b: 1.5 points

Sample 1
Limited reported losses = 392,457 – 128,305 + 1,013,863 – 525,626 – 152,860 + 459,798 – 275,865 + 400,000
= 1,183,462

Exp Mod = $1+z\left(\frac{A}{E}-1\right)$
= $1 + 0.4\left(\frac{1{,}183{,}462}{975{,}763}-1\right)$
= 1.085

Unlimited Expected Loss = 1,064,000 * 1.085 = 1,154,653

Sample 2

PY	Total Capped Rpt Loss
2016	392,457 – 128,305 + 100,000 = 364,152
2017	1,013,863 – 525,626 – 152,860 + 100,000*2 = 535,377
2018	459,798 – 275,865 + 100,000 = 283,933
Total	= 1,183,462 = expected gnd up reported loss capped at 100k

$Z = \frac{E}{E+K}$
$ZE + ZK = E$
$\frac{E(1-Z)}{Z} = K$
$Z = 0.4$
$E = 974{,}644$ (from a)
-> $K = 1{,}461{,}966$

Mod = $\frac{A+K}{E+K} = \frac{1{,}183{,}462+1{,}461{,}966}{974{,}644+1{,}461{,}966}$
Mod = 1.0857

Total modified gnd up ult loss
=1.0857 * 1,064,000
=1,155,184

Alternate Solution
Actual reported loss limited to $100,000 per claim

Year	Actual Loss Limited	
2018	459,798 - (275,865 - 100,000)	283,933
2017	1,013,863 – (525,626 - 100,000) – (152,860 – 100,000)	535,377
2016	392,457 – (128,305 - 100,000)	364,152
	Total=	1,183,462

Mod = (Actual/Expected) *z + (1.0)*(1-z)
= (1,183,462/2,457,841)(0.40) + (1-0.40)
=0.7926

Modified groundup limited ult loss = (1,064,000)(0.7926)
=843,539

E[X; 100,000] = $\frac{22{,}800}{1.3-1}\left(1-\left(\frac{22{,}800}{100{,}000+22{,}800}\right)^{1.3-1}\right)$ = 30,139.95
E[X] = $\frac{22{,}800}{1.3-1}$ = 76,000
$\frac{30{,}139.95}{76{,}000}$ = 0.397

Modified ground up unlimited expected loss
= $\frac{843{,}539}{0.397}$ = 2,124,783

Part c: 0.25 point

Sample Responses

- This approach would give equal weight to all years of experience while the method we used in this exercise gives more weight to older years' experience
- By trending historical loss and keeping the expected at ultimate level, we're assigning equal weights to the expected loss. From part a, the detrended, un-developed losses do not have equal weights therefore the mod will be different between the methods.
- Using reported loss puts more weight on older years

Part d: 0.5 point

Sample 1

GCP = $\frac{E[L]*LAE\%}{1-V}$ = $\frac{1.064M*1.085*1.075}{1-0.035-0.17}$ = $1.561M

Sample 2

GCP = (e+E)T

e = 0.075 * 1,154,563 = 86,592

T = $\frac{1}{1-.035-.17}$ = 1.258

GCP = (86,592 + 1,154,563) * 1.258 = $1,561,373

Part e: 1.5 points

Sample 1:

ELG = 28 based on E[X] = 1,155,078

R_g = 2M/1,155,078 ≈ 1.75

R_h = 4M/1,155,078 ≈ 3.50

$E[X\}_r$ = 1,155,078 * (0.1131 – 0.0148)

= 113,544

Sample 2:

E[Loss] = 1,155,070

ELG = 28

R_{high} = 4,000,000/1,155,070 = 3.46

R_{low} = 2,000,000/1,155,070 = 1.73

$\phi_{3.46}$ = .0148

$\phi_{1.73}$ = .1131

so loss in layer = (.1131 - .0148) * 1,155,070

= 113,543.38

Part f: 1 point

Sample 1:

$$\frac{1{,}155{,}078 - 113{,}544 + 1{,}155{,}078 * (0.075) + 200{,}000}{1 - .035 - .017}$$

= 1,670,648

Sample 2:

P= 200k
Diff = 200k-113,546 =86,454
GCP from d. = 1,561,827
Primary insurer must charge:

$$1{,}561{,}927 + \frac{86{,}454}{1 - .035 - .017}$$

= 1,670,674

Sample 3:

$$\frac{(1{,}154{,}550) * (1 + 7.5\%) - 113{,}492 + 200{,}000}{1 - 3.5\% - 17\%}$$

= 1,669,999

Part g: 0.5 point

Sample 1:
The assumption of a Pareto distribution may not be accurate which will have a large impact on the tail of the distribution. This could make the insurance charges under or overstated making for an inequitable premium.

Sample 2:
The calculation in part f doesn't account for the primary insurer transferring a large portion of uncertain risk to the reinsurer. To account for this, the primary insurer could lower its profit or risk load.

EXAMINER'S REPORT

Candidates were expected to understand the actuarial principles and concepts underlying the development of experience rating plans. They were expected to demonstrate this knowledge by calculating the experience modification factor for a policy and the subsequent modified premium. Many candidates were unclear as to how the loss cap impacts the experience mod and the prospective premium.

Candidates were then expected to use a Table M lookup to determine the ceded loss and needed premium under a reinsurance treaty, and to comment on the impact of actuarial assumptions on these values.

Candidates should note that parts a through d of this question were very similar to steps 2, 3 and 4 of the Exam 8 Syllabus Case Study.

Part a

Candidates were expected to be able to bring prospective expected losses to the levels in the loss experience period on a limited basis in order to facilitate the calculation of an experience mod. Candidates needed to determine the correct trend period, calculate the appropriate loss

elimination ratio (i.e. E[X; x] / E[X]) using the shifted pareto distribution, and apply the appropriate limited loss development factors.

Answers that assumed $1,064,000 was a *limited* ultimate expected loss were also accepted if the candidate calculated the appropriate ILF to use in part b.

Common mistakes included:

- Not calculating a loss elimination ratio
- Using reported losses to calculate a loss elimination ratio
- Applying an excess ratio instead of a loss elimination ratio
- Using F(100k) as a loss elimination ratio
- Using the limited expected severity as the ultimate loss
- Detrending the $100k loss cap to the experience period
- Adjusting the reported losses (e.g. trending and developing to the prospective policy period)
- Using the wrong trend period
- Assuming the given loss development factors were age-to-age factors
- Using 4% trend instead of 4.5%.

Part b

Candidates were expected to apply the individual loss cap to reported losses, select the appropriate policy years, calculate the experience modification using the credibility factor, and calculate the final unlimited expected loss.

Answers that assumed $1,064,000 was a limited ultimate expected loss were also accepted, but required the candidate to also calculate the appropriate increased limits factor using the shifted pareto distribution.

Candidates who skipped the mod calculation completely and calculated a credibility-weighted answer using unlimited expected and reported losses received very little credit.

Common mistakes included:

- Using unlimited reported and/or expected values to calculate the mod
- Trending or developing reported losses
- Dividing reported losses by the prospective expected loss
- Forgetting to calculate a final expected loss after calculating the mod.

Part c

Candidates were expected to describe a reason why trending and developing reported losses to the cost level of the prospective policy period may not produce an identical experience mod to the method calculated in part b.

Many candidates stated that the experience mod would differ due to the interaction of the loss trend and the loss cap. This answer was not accepted because the interaction is also contemplated when detrending and undeveloping losses to the experience period.

Common mistakes included:

- Stating that the methods would differ due to the loss cap
- Stating that projecting reported losses would be a less credible method
- Stating that reported losses may differ from the assumed shifted Pareto distribution.

Part d

Candidates were expected to calculate the final guaranteed cost premium using the experience-modified unlimited expected losses, the LAE %, and the other variable expense components.

Common mistakes included:

- Ignoring the effect of the experience mod when calculating the expected loss
- Assuming the policy was a large deductible or retrospectively rated policy
- Using limited losses, or otherwise assuming a prospective policy limit of $100k
- Using reported losses as expected loss
- Using the wrong LAE load (7% vs 7.5%).

Part e

Candidates were expected to use the modified ground up unlimited expected loss as calculated in part b and the parameters of the reinsurance treaty to determine the ELG, entry ratios, and table M charges. Using the difference between the table M charges and the modified ground up unlimited expected loss, they then calculated the loss expected to be ceded to the reinsurer. Any answer from part b., regardless of rounding or calculation errors, could be pulled forward to part e. and potentially receive full credit.

Candidates received small deductions when using the expected ultimate loss before modification, even if they stated that as their assumption. Candidates were not penalized for assuming the closest ELG or table M charge when their calculated loss or entry ratio fell outside the given tables.

Common mistakes included:

- Using the unmodified loss
- Using modified losses to determine the ELG, but not the entry ratios or vice versa.
- Calculating insurance savings instead of insurance charge
- Attempting to interpolate values instead of rounding to the nearest entry ratio.

Part f

Candidates were expected to use the expected ceded loss calculated in part e to calculate the new premium the primary insurer must charge. Most successful candidates calculated the retained losses, LAE, and reinsurer premium as shown in Sample 1. Another common response was to calculate the additional premium needed due to reinsurance and add that number to the guaranteed cost premium from part d as shown in Sample 2. Candidates could calculate a mathematically equivalent answer without directly calculating retained losses, which is shown in Sample 3. All correct solutions maintained the 0% profit provision, treated the reinsurance as a fixed expense, and accounted for variable expenses. Any answer from part b., d. and/or e., regardless of rounding or calculation errors, could be pulled forward to part f. and potentially receive full credit.

Common mistakes included:

- Calculating a profit provision for the reinsurer, and then calculating a new premium for the primary insurer using that provision.
- Failing to correctly adjust for variable expenses.
- Calculating only the additional premium the primary insurer should charge for the reinsurance treaty, but not the total premium.

Part g

Candidates were expected to reference one of several assumptions the actuary made in this problem and describe how that assumption would affect the premium if incorrect. The most common correct answers referenced the shifted Pareto claim severity or the assumption that the reinsurer and primary insurer should maintain the same underwriting profit despite the transfer of a risky layer to the reinsurer.

Candidates generally did not receive full credit for creating an assumption not mentioned in the problem and then stating that it wasn't met. For example, a candidate might state that the given Table M had not been adjusted for the loss limitations, and then state that this was inappropriate. This question was challenging for most candidates.

Fisher et. al. (p.96) discusses the potential mismatch of the distributions underlying the expected loss and aggregate charges as a source of error. Clark (p. 42) discusses how the parameter variance of the aggregate distribution can lead to errors. A discussion of either of these responses would have been acceptable, full credit answers.

Common mistakes included:

- Stating that a 0% profit provision was inherently inequitable, which is not true due to investment income.
- Stating that it was unreasonable for the reinsurance premium to be treated as fixed.