13. (2 points)

For a given policy that provides first dollar coverage, there is a 50% probability that no claims will occur during the policy period. Given that a claim does occur, the frequency distribution is shown below:

Number of Claims	Probability
1	70%
3	20%
5	10%

The claim-size variable X for this policy follows a shifted Pareto distribution with the following characteristics:

- $e_X(6{,}000) = 15{,}000$
- $E[X] = 3{,}000$
- $E[X; 4{,}000] = 1{,}433.30$

a. (1.5 points)

Calculate the expected aggregate loss in the layer 1,000 excess of 4,000 for this policy.

b. (0.5 point)

Explain one difficulty that can arise when trying to fit an aggregate loss distribution function to losses in an excess layer.

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QUESTION 13	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): B2**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

Known Information

$E[S] = E[N](E[X;5000] - E[X;4000])$

$E[X;Y] = \frac{\beta}{\alpha-1}\left(1-\left(\frac{\beta}{Y+\beta}\right)^{\alpha-1}\right) \quad \rightarrow \quad E[X;4000] = \frac{\beta}{\alpha-1}\left(1-\left(\frac{\beta}{4000+\beta}\right)^{\alpha-1}\right)$= 1433.30

$E[X] = \frac{\beta}{\alpha-1} = 3000 \quad \rightarrow \quad \beta = 3000(\alpha-1)$

$e_x(x) = \frac{x+\beta}{\alpha-1} \quad \rightarrow \quad e_x(6000) = \frac{6000+\beta}{\alpha-1} = 15000$

Find β and α

$\frac{6000+\beta}{\alpha-1} = 15000$

$6000+\beta = 15000(\alpha-1)$

$6000 + 3000(\alpha-1) = 15000(\alpha-1)$

$6000 = 12000(\alpha-1)$

$\alpha = 1.5 \qquad \beta = 3000(\alpha-1) = 1500$

Find the value of $E[X;5000]$

$E[X;5000] = \frac{\beta}{\alpha-1}\left(1-\left(\frac{\beta}{5000+\beta}\right)^{\alpha-1}\right) = \frac{1500}{1.5-1}\left(1-\left(\frac{1500}{5000+1500}\right)^{1.5-1}\right) = 1558.85$

Find the value of $E[N]$

$E[N] = 0.5 \times 0 + 0.5 \times (0.7 \times 1 + 0.2 \times 3 + 0.1 \times 5) = 0.9$

Find the value of $E[S]$

$E[S] = E[N](E[X;5000] - E[X;4000]) = 0.9 \times (1558.85 - 1433.30) = 112.995$

Sample 2

Average excess claim size in layer 1000 xs 4000

$$= \frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)}$$

Since $E[X] = \frac{\beta}{\alpha-1}$, $\quad E[X;Y] = \frac{\beta}{\alpha-1}\left(1-\left(\frac{\beta}{Y+\beta}\right)^{\alpha-1}\right)$, $\quad e_x(x) = \frac{x+\beta}{\alpha-1}$, $\quad F_x(x) = 1-\left(\frac{\beta}{x+\beta}\right)^{\alpha}$

$\frac{\beta}{\alpha-1} = 3000$, $\frac{6000+\beta}{\alpha-1} = 15000 \qquad \alpha = 1.5 \ \ \beta = 1500$

$$\frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)} = \frac{(1558.85 - 1433.3)}{1 - 0.8576} = 881.5$$

Expected number of claims

$= 0.5 \times (1 \times 0.7 + 3 \times 0.2 + 5 \times 0.1) = 0.9$

Expected number of claims in layer

$= 0.9 \times \left(1 - F_x(4000)\right) = 0.1282$

Expected aggregate loss in layer

$= 881.5 \times 0.1282 = 113$

Sample 3

$$E[X] = \frac{\beta}{\alpha-1}, \quad E[X;Y] = \frac{\beta}{\alpha-1}\left(1-\left(\frac{\beta}{Y+\beta}\right)^{\alpha-1}\right),$$
$$e_x(x) = \frac{x+\beta}{\alpha-1}, \quad F_x(x) = 1-\left(\frac{\beta}{x+\beta}\right)^{\alpha}$$

$\frac{\beta}{\alpha-1} = 3000$, $\frac{6000+\beta}{\alpha-1} = 15000 \qquad \alpha = 1.5 \ \ \beta = 1500$

Expected number of claims

$= 0.5 \times (1 \times 0.7 + 3 \times 0.2 + 5 \times 0.1) = 0.9$

Expected number of claims in layer

$= 0.9 \times \left(1 - F_x(4000)\right) = 0.1282$

Expected aggregate loss in layer

$$= \frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)} \times 0.9 \times \left(1 - F_x(4000)\right) = 0.9 \times (E[X;5000] - E[X;4000])$$
$$= 0.9 \times (1558.85 - 1433.3) = 112.995$$

Part b: 0.5 point

- The aggregate layer has fewer observations (loss experience) to fit, thus data is very volatile and thin, which makes it harder to estimate. And it can be difficult to smooth the

layers between lower and higher areas to have a good smoothed transition when using the fitted curve.
- Excess losses tend to be sparse and may not be credible enough to give a consistent distribution fitting without some jump discontinuities.

EXAMINER'S REPORT

Candidates were expected to understand how to combine frequency and severity distributions into an aggregate loss distribution and to calculate the expected aggregate loss for a specified layer. Candidates were then expected to demonstrate an understanding of the realities of approximating an aggregate distribution with losses in an excess layer.

Part a

Candidates were expected to evaluate the α and β parameters of the severity distribution from the given information, before calculating the expected severity in the excess layer under consideration and combining with the unconditional expected claim count.

Common mistakes included:
- Incorrectly calculating the parameters α and β
- Using the conditional instead of unconditional expected claim count (i.e. forgetting to adjust for the 50% probability of 0 claims)
- Calculating the expected severity for the incorrect layer.

Part b

Candidates were expected to recognize the particularities of excess layers and to identify the small expected claim count as one of the drivers resulting in jump discontinuities in the cumulative distribution function.

Common mistakes included:
- Merely specifying that data is volatile, without reference to claim count
- Alluding to the required independence of frequency and severity distributions
- Neglecting to relate the small expected claim count to the resulting jump discontinuities