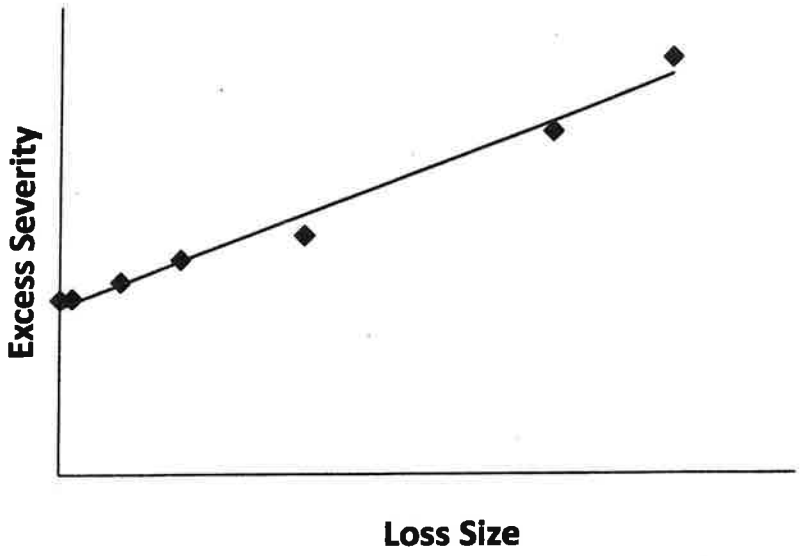
15. (3.5 points)

An actuary is pricing a retrospectively rated policy with a per occurrence limit of 20,000 and no minimum or maximum ratable loss.

Using claim data from a group of similar policies, the actuary fit the following regression line to the excess severity to estimate losses using the shifted Pareto distribution. The fitted regression function is $y = 0.4467x + 16112$ with $R^2 = 0.9826$.



Given the following:

		Included in:
Loss adjustment expenses as a percentage of loss	10%	Loss conversion factor
Commission as a percentage of retrospective premium	9%	Tax multiplier
Premium tax as a percentage of retrospective premium	8%	Tax multiplier
Fixed overhead expenses	50,000	Basic premium
Underwriting profit as a percentage of expected excess loss	6%	Basic premium

- The number of claims for ground up losses is Poisson distributed with $\lambda = 15$.

CONTINUED ON NEXT PAGE

a. (2 points)

Calculate the basic premium.

b. (1 point)

Assuming that actual limited losses equal half the expected limited losses, calculate the retrospective premium.

c. (0.5 point)

Assess the actuary's decision to use the shifted Pareto distribution to estimate excess severity.

CONTINUED ON NEXT PAGE

QUESTION 15	
TOTAL POINT VALUE: 3.5	**LEARNING OBJECTIVE(S): B1, B6**

SAMPLE ANSWERS

Part a: 2 points

Sample 1

Slope = 1 / (α – 1) = .4467
Intercept = β / (α – 1) = 16,112
B = 36,069
α = 3.24

E[X] – E[X;20,000] = $(\frac{36,069}{3.24-1})$ (1 - $(\frac{36,069}{20,000+36069})^{3.24-1}$ = 10,108
E[X] = 16,102

Aggregate
E[A] = (15)(16,112) = 241,530
$E[A_D]$ = (15)(10,108) = 151,620
$E[A_e]$ = 241,530 - 151,620 = 89,910

B = 50,000 + 1.06(89,910) + (89,910)1.1=154,296

Sample 2

y = 0.4467*20000 + 16112 = 25,046
e(20,000) = (20K + β) / (α – 1) = 25,046
e(0) = 16112 = β / (α-1)
16112(α-1) = β
(20000 + 16112(α-1)) / (α-1) = 20000/(α-1) + 16112 = 25046
α = 3.239
β = 36.068.95
S(20K) = $(\frac{36068.95}{20000+36068.95})^{3.239}$ = 0.2396
Excess loss = 25046*15*S(20K) = 25046 * 3.594 = 90,015
Basic Premium = 50,000 + 0.06*90015 + 1.1*90015 = 154,417.77

Part b: 1 point

Sample 1

Expected severity below limit = $(\frac{36068.95}{3.239-1})(1-(\frac{36068.95}{20000+36068.95})^{3.239-1})$ = 10,109.95
Expected losses below limit = 10,109.95 * 15 = 151,649
Actual losses below limit = 151,649 / 2 = 75,825

Solve for Retro premium:
= (basic premium + LCF*actual loss below limit)/(1-commission-tax)
= (154,417.77 + 1.1*75,825)/(1-0.09-0.08)
= 286,536

Part c: 0.5 point

Sample 1

Yes, R^2 is high and the fitted line is close to the observed values, indicating that the line fits well. However, the slope of the regression line is sensitive to the size of the largest claims, and the calculated distribution parameters could be significantly affected by changes in just a few of these numbers.

Sample 2

The regression line is linear and upward sloping which matches the shape of a typical Pareto excess severity distribution. Further, the fitted line is reasonably close to the actual data and the quality of the fit is also confirmed by a high R^2 value.

EXAMINER'S REPORT

Candidates were expected to demonstrate understanding of an excess severity function and the components of a retrospective rated policy, including the relationship between them.

Part a

Candidates were expected to translate the excess severity function given into the alpha and beta parameters of a pareto distribution. Using that information, candidates were then expected to calculate the expected excess losses, and to use the expense assumption given to determine the basic premium.

If candidates were unable to solve for alpha and beta and instead made an assumption of their values, partial credit was given if subsequent calculations were correct.

Some candidates did not include excess losses in the basic premium calculation. However, if these candidates later included a separate excess loss premium in their calculation of the retro premium in part b, then full credit was given for part a.

Common mistakes included:

- Not solving for alpha or beta
- Mistaking the fixed overhead expenses of $50,000 for e, the expenses underlying a guaranteed cost premium
- Not multiplying the expected excess severity by the expected frequency in the excess loss calculation
- Forgetting to adjust by the profit load
- Not applying the LCF to expected excess loss
- Using expected limited loss instead of excess loss in the basic premium calculation
- Not including excess losses in either the basic premium or retrospective premium.

Part b

Candidates were expected to use the alpha and beta parameters calculated in part a to calculate the expected losses below the limit (or use the value of limited expected losses from part a if

already calculated). Candidates were then expected to use the information provided in the problem to calculate actual losses from expected losses below the limit. Candidates who used an incorrect value for expected losses below limit (calculated in part a) were given credit in part b if they correctly calculated actual losses by dividing this value by 2.

Candidates were then expected to calculate the final retrospective premium by using the actual limited losses in the retro premium formula, which includes the basic premium, converted actual losses, and the tax multiplier.

Commons mistakes included:

- Using unlimited expected losses above the limit to calculate actual losses
- Not including frequency in the actual loss calculation
- Not applying the LCF to actual limited loss
- Incorrectly calculating the tax multiplier.

Part c

Candidates were expected to comment on the actuary's decision to fit a regression line to excess severity to estimate losses using the shifted Pareto distribution. This included commenting on the fit of the line vs. the plotted points, the R^2 value included in the question, and/or the shape of the fitted line.

Candidates were also expected to discuss the sensitivity of the slope of the regression line to the size of the largest claims, and how the distribution parameters could be impacted by a change in any of these larger losses. Candidates needed to make appropriate comments that discussed the overall quality of either the choice of distribution, the fit of the distribution, and/or the sensitivity of the fit to the size of the largest claims.

Common mistakes included:

- Incorrectly stating the shape of the fitted line did not match the shape of the Pareto distribution when fitting a regression line to excess losses
- Stating the R^2 value was low and not indicative of a good fit
- Not offering enough detail to assess the actuary's decision.