Reading: Fisher.Visualization
Model: Source text
Problem Type: Derive the balance equations for a retrospective rating plan

Fisher_BalEqDeriv

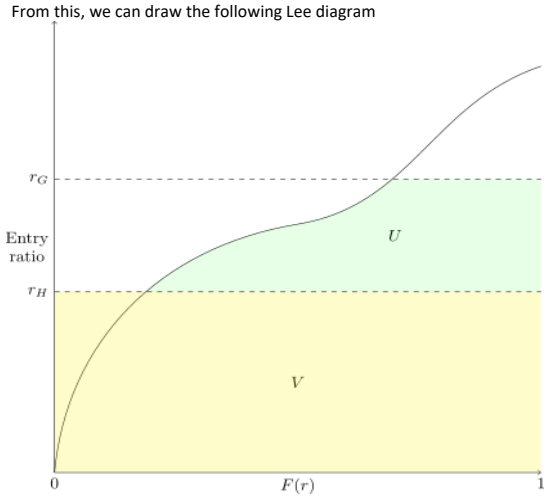
A retrospective rating plan has a base premium, B, that is fixed (doesn't vary with loss).

A retrospective rating plan also has a minimum premium H and a maximum premium G.
These correspond to a minimum ratable loss L_H and a maximum ratable loss L_G respectively.

Using the retrospective rating formula, $R = (B + c \cdot L)T$
and ignoring the tax multiplier by setting T = 1, we get $H = B + c \cdot L_H$ and $G = B + c \cdot L_G$

Letting E be the expected loss gives the following entry ratios: $r_H = \frac{L_H}{E}$ and $r_G = \frac{L_G}{E}$

From this, we can draw the following Lee diagram



The green and yellow area is the average ratable loss, so $\frac{L}{E} = U + V$ [1]

Since the premium for a retrospective rating plan should cover the expected loss and expenses we have $R = e + E$
Equating this with the retrospective rating formula (with T=1) gives

$$e + E = B + c \cdot (U + V) \cdot E \quad [2]$$

where we used [1] to replace L

At the minimum premium, H, the area of U is zero so we get

$$H = B + cL_H = B + cVE$$

which rearranges to $H - B = cVE$ [3]

Substituting [3] into [2] yields $e + E = cUE + H$ [4]

However, from the Lee diagram we know $U = \phi(r_H) - \phi(r_G)$ [5]

Substituting [5] into [4] yields $e + E = c \cdot \big(\phi(r_H) - \phi(r_G)\big) \cdot E + H$

which then rearranges to the first balance equation: $\boldsymbol{\phi(r_H) - \phi(r_G) = \frac{e + E - H}{c \cdot E}}$

Next, ratable losses associated with the minimum premium may be expressed as $L_H = r_H \cdot E$
Applying the retrospective rating formula yields $H = B + c \cdot r_H \cdot E$
Similarly we have $G = B + c \cdot r_G \cdot E$

Taking the difference of these two equations gives $G - H = c \cdot (r_G - r_H) \cdot E$

which rearranges to the second balance equation: $\boldsymbol{r_G - r_H = \frac{G - H}{c \cdot E}}$

Note:
The first balance equation tells us $\phi(r_H) - \phi(r_G)$
(the green shaded area labelled by U) is the difference between the expected retrospective premium at the minimum ratable loss and the minimum premium (since R = e + E), scaled by the factor cE.