Reading: Fisher.RiskSharing Fisher_CashflowRetro (Problem 1)
Model: Source Text
Problem Type: Visualize the cash flow for both the policyholder and the insurer under an incurred retrospective rating plan.

Given

Pricing Assumptions	
$1,100,000	Initial Premium
$600,000	Expected Primary Loss & ALAE
$300,000	Expected Excess Loss & ALAE
$55,000	Commission
$15,000	General Expenses
$5,000	Underwriting Profit Provision
10.0%	ULAE
3.0%	Tax Rate

There is no aggregate excess loss exposure.

Payment Patterns

Time (Years)	Initial Premium	(1) Primary Incurred Loss & ALAE	(2) Primary Paid Loss & ALAE	(3) Excess Paid Loss & ALAE	(4) Total Paid Loss & ALAE	Commission	(5) General Expenses	(6) ULAE
0.00	100%					100%	25.0%	
0.25		10.7%	2.1%	0.1%	1.4%		43.8%	7.3%
0.50		26.3%	7.2%	0.5%	5.0%		62.5%	16.2%
0.75		45.4%	14.5%	2.0%	10.3%		81.3%	26.5%
1.00		65.5%	23.4%	5.0%	17.3%		100.0%	38.0%
1.50		77.3%	40.9%	15.0%	32.3%			49.2%
2.50		87.9%	63.5%	35.0%	54.0%			65.5%
3.50		93.9%	79.8%	60.0%	73.2%			79.9%
4.50		97.4%	90.4%	80.0%	86.9%			90.2%
5.50		98.9%	95.6%	90.0%	93.7%			95.3%
6.50		99.7%	97.7%	95.0%	96.8%			97.6%
7.50		100.0%	100.0%	100.0%	100.0%			100.0%

Find The incurred retrospective rating plan basic premium at each point in time and illustrate the cash flows from both the policyholder and insurer perspectives.

Note As the CAS moves towards computer based testing, this type of problem (which is tedious to do by hand) becomes much easier to test.

Solution

The loss conversion factor is c = 1 + ULAE %, i.e.

c = 1.100

Now let's calculate the basic premium which should cover the converted expected excess loss and ALAE along with any fixed expenses.

The basic premium is the (Expected Excess Loss & ALAE multiplied by the loss conversion factor) plus commission, general expenses, and UW profit.

B = $300,000 * (1 + 10.0%) + $55,000 + $15,000 + $5,000

= $405,000 **<= Basic Premium**

The tax multiplier, T, is 1 / (1 - 3.0%)

T = 1.031

To calculate the incurred retrospective rating premium we need L, the ratable loss amount.
We'll use the payment pattern to determine it at each point in time.
We're given the payment pattern, let's look at this information in more detail before working with it.
1. This is a 1-year incurred retrospective rating plan; no premium adjustments will occur until 18 months have elapsed, and then are evaluated annually.
2. The initial premium is paid immediately at the start, along with the commission.
3. We assume all losses are at ultimate after 7.5 years and that ALAE is included in the ratable loss.
4. Since it's a 1-year policy, all general expenses happen within the first year. ULAE is accrued all the time the losses aren't at ultimate.

Policyholder Cash Flow

Time (Years)	(7) Primary Incurred Loss & ALAE [Ratable Loss, L]	(8) Total Premium Paid	(9) Cumulative Cash Flow	(10) Incremental Cash Flow	Comments
0.00	0	$1,100,000	($1,100,000)	($1,100,000)	
0.25	$64,200	$1,100,000	($1,100,000)	$0	
0.50	$157,800	$1,100,000	($1,100,000)	$0	
0.75	$272,400	$1,100,000	($1,100,000)	$0	
1.00	$393,000	$1,100,000	($1,100,000)	$0	
1.50	$463,800	$943,551	($943,551)	$156,449	First premium adjustment occurs at t = 1.5. Losses better than expected so
2.50	$527,400	$1,015,679	($1,015,679)	($72,128)	the policyholder receives a partial premium refund from the insurer.
3.50	$563,400	$1,056,507	($1,056,507)	($40,828)	
4.50	$584,400	$1,080,323	($1,080,323)	($23,816)	Subsequent evaluations (t ≥ 2.5) show losses gradually deteriorating;
5.50	$593,400	$1,090,530	($1,090,530)	($10,207)	this requires additional premium payments to the insurer.
6.50	$598,200	$1,095,974	($1,095,974)	($5,444)	
7.50	$600,000	$1,098,015	($1,098,015)	($2,041)	

(7) = (1) * Expected Primary Loss & ALAE
(8) This is the initial premium until 1.5 years have elapsed. Afterwards, use (8) = [B + c * (7)] * T
(9) = -1 * (8) as these are the cumulative payments made by the policyholder.
(10) = [(9) current row] - [(9) prior row]

The ratable loss (column 7) is the primary incurred loss and ALAE after the consideration of any maximum or minimum ratable loss.

The requirement for the insured to make additional premium payments after the end of the policy period creates credit risk for the insurer.

Note:

In the text, Fisher uses an unrounded value of T. Here we've rounded T to 3 decimal places for convenience.

On the next page we look at the cash flow from the insurer's perspective.

Insurer Cash Flow

Fisher_CashflowRetro (Solution 1-2)

Columns (11) – (19) are cumulative figures to date

Time (Years)	(11) Premium	(12) Primary Loss & ALAE Paid	(13) Excess Loss & ALAE Paid	(14) Total Loss & ALAE Paid	(15) Total Commission	(16) Premium Tax	(17) General Expenses	(18) ULAE	(19) Cash Flow	(20) Incremental Cash Flow
0.00	$1,100,000	$0	$0	$0	$55,000	$33,000	$3,750	$0	$1,008,250	$1,008,250
0.25	$1,100,000	$12,600	$300	$12,900	$55,000	$33,000	$6,570	$6,570	$985,960	($22,290)
0.50	$1,100,000	$43,200	$1,500	$44,700	$55,000	$33,000	$9,375	$14,580	$943,345	($42,615)
0.75	$1,100,000	$87,000	$6,000	$93,000	$55,000	$33,000	$12,195	$23,850	$882,955	($60,390)
1.00	$1,100,000	$140,400	$15,000	$155,400	$55,000	$33,000	$15,000	$34,200	$807,400	($75,555)
1.50	$943,551	$245,400	$45,000	$290,400	$55,000	$28,307	$15,000	$44,280	$510,564	($296,836)
2.50	$1,015,679	$381,000	$105,000	$486,000	$55,000	$30,470	$15,000	$58,950	$370,259	($140,306)
3.50	$1,056,507	$478,800	$180,000	$658,800	$55,000	$31,695	$15,000	$71,910	$224,102	($146,157)
4.50	$1,080,323	$542,400	$240,000	$782,400	$55,000	$32,410	$15,000	$81,180	$114,333	($109,768)
5.50	$1,090,530	$573,600	$270,000	$843,600	$55,000	$32,716	$15,000	$85,770	$58,444	($55,889)
6.50	$1,095,974	$586,200	$285,000	$871,200	$55,000	$32,879	$15,000	$87,840	$34,055	($24,389)
7.50	$1,098,015	$600,000	$300,000	$900,000	$55,000	$32,940	$15,000	$90,000	$5,075	($28,980)

(11) = (8)
(12) = (2) * Expected Primary Loss & ALAE
(13) = (3) * Expected Excess Loss & ALAE
(14) = (12) + (13)
(15) Commission is paid upfront then doesn't change.
(16) = (11) * 3.0% (Premium tax rate)
(17) = (5) * General Expenses
(18) = (Expected Primary and Excess Loss & ALAE) * ULAE % * (6)
(19) = (11) - (14) - (15) - (16) - (17) - (18)
(20) = [(19) current row] - [(19) prior row]

Notes:

- Both the policyholder and insurer have negative cash flows after t = 2.5. The insurer has a negative cash flow because it is paying out on losses. The policyholder has a negative cash flow because they exchanged a larger upfront premium (guaranteed cost premium) for a lower initial premium with additional premium payments later and the potential to receive premium refunds if their experience was better than expected.
- The additional premium payments from the policyholder adjust but do not completely offset the loss experience paid by the insurer at the time.
- The final cash flow figure of $5,075 is a result of rounding the premium tax, T, to 3 decimal places. If the full precision is used then we are left with exactly the UW profit provision of $5,000.