Reading: Fisher.TableL

Fisher_Ch3Q14 (Problem 1)

Model: Source text: Chapter 3 Q14

Problem Type: Draw a Lee diagram and calculate the Table L insurance charge and savings

Given A policy has the following properties:

- Its unlimited loss distribution is continuous and uniform on the interval [0, 500]
- Its limited loss distribution is continuous and uniform on the interval [0, 400]
- Its entry ratio is 1.5 times the expected unlimited loss.

Find Draw a Lee diagram representing this policy and calculate the following:

a) $\phi(1.5)$

b) $\varphi(1.5)$

Solution Fisher_Ch3Q14 (Solution 1)

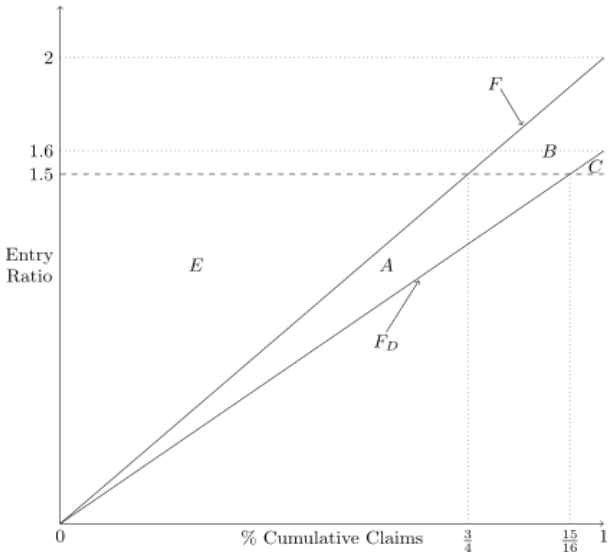
First we need to know the maximum entry ratio for the unlimited distribution. The unlimited loss distribution has an expected loss of 250
So its maximum entry ratio is 500 / 250 = 2.
Similiarly, we get the minimum entry ratio for the unlimited distribution as 0 / 250 = 0
Since the unlimited loss distribution is continuous and uniform, we know its Lee diagram will be a straight line from (0,0) to (1,2)

Next, we need to plot the limited loss distribution. Recall the formula for the Table L entry ratio is

$$\frac{\text{Limited Aggregate Loss}}{\text{Expected Unlimited Aggregate Loss}}$$

Since the limited loss distribution is continuous and uniform, we know it will be represented by a straight line.
The minimum entry ratio for the limited loss distribution is 0 / 250 = 0
The maximum entry ratio for the limited loss distribution is 400 / 250 = 1.6

Bringing this all together yields the following Lee diagram



Alice: "On a side note there are two ways you can figure out the corresponding x coordinate for any given entry ratio.
First, you could find the equation of the line through (0,0) and (1,1.6) and then solve for x after substituting in the desired entry ratio for y.
The second way is to set the known entry ratio equal to the Table L entry ratio definition. Using an entry ratio of 1.5 as an example we have 1.5 = (Limited Actual Loss) / (Expected Unlimited Loss).
*We know the expected unlimited loss is 250, so the limited actual loss must be 1.5 * 250 = 375.*
Now, recalling we're interested in curve F_D, the maximum possible limited loss is 400. So the associated x value is 375/400 = 1.5/1.6."

From the Lee diagram we can deduce the areas which represent the Table L insurance charge and savings at an entry ratio of 1.5

Table L insurance charge = A + B + C
Table L insurance savings = A + E

Note the area under the curve F is equal to 1 and we can calculate x-axis coordinates by taking the ratio of the entry ratio to the maximum entry ratio for each curve (see Alice's sidenote). This gives

ϕ(1.5) = 1 - 0.5*1*1.6 + 0.5*(1 - 1.5 / 1.6) * (1.6 - 1.5) = 0.203125

Then using $\psi_D^* = \phi_D^*(r) + r - 1$

φ(1.5) = 0.203125 + 1.5 - 1 = 0.703125