Reading: Fisher.AggExcess
Model: Source text
Problem Type: Calculate the Table M insurance savings from first principles given an exponential aggregate loss distribution.

Given

A ~ Exponential — Aggregate loss distribution
E[A] = 10

Find

Calculate the Table M Savings and insurance savings for the following actual losses.

A	Table M Savings	Insurance Savings
5	?	?
10	?	?
15	?	?

Solution

First we need the pdf and cdf for an exponential distribution with mean Θ

p.d.f. $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ c.d.f $F(x) = 1 - e^{-\frac{x}{\theta}}$

Next, we need the formula for the Table M Savings: $\psi(r) = \int_0^r (r-y)\mathrm{d}F(y)$

It's clear we're going to need to work with entry ratios instead of actual and expected losses.
Remember, the entry ratio is just the actual loss divided by the expected loss.

Form the new distribution: $Y = \frac{A}{E}$ *Alice: "This is a really important part - forming the correct distribution."*

We now need the p.d.f. and c.d.f. of the new distribution. To do this, it's helpful to recall the following relationship:

Let $Y = aX + b$ where a is non-zero. Then the p.d.f. of Y is given by $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$

Here $f_Y(y)$ is the p.d.f. of Y and $f_X(x)$ is the p.d.f. of X.

Using the above, our exponential distribution A which has mean 10, yields a = 1 / 10. So the p.d.f. of Y is:

$$f_Y(y) = \frac{1}{\left(\left|\frac{1}{10}\right|\right)} \cdot \frac{1}{10} e^{-\frac{\left(\frac{y}{\left(\frac{1}{10}\right)}\right)}{10}} = e^{-y}$$

This is an exponential distribution with mean 1 and so $F(y) = 1 - e^{-y}$.

Plugging this into the formula for the Table M Savings gives: $\psi(r) = \int_0^r (r-y) \cdot e^{-y} \mathrm{d}y$

By carefully evaluating this integral, we can complete the table as follows:

A	Entry Ratio	Table M Savings	Insurance Savings
5	0.5	0.1065	1.0653
10	1.0	0.3679	3.6788
15	1.5	0.7231	7.2313