Reading: Fisher.Visualization Fisher_LtdTblMBalEqDeriv
Model: Source text
Problem Type: Derive the Limited Table M balance equations for a balanced retrospective rating plan

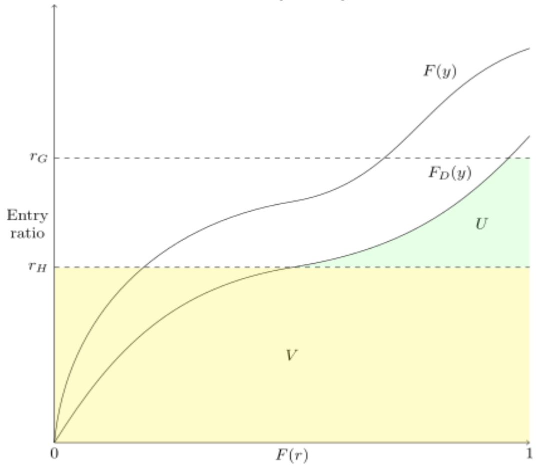
A retrospective rating plan has a basic premium, B, that is fixed (doesn't vary with loss). Assume the plan only has a per-occurrence limit/deductible.

We'll also assume our retrospective rating plan also has a minimum premium H and a maximum premium G.
These correspond to a minimum ratable loss L_H and a maximum ratable loss L_G respectively.

Using the retrospective rating formula, $R = (B + c \cdot L) \cdot T$ [1]
we get $H = (B + c \cdot L_H) \cdot T$ and $G = (B + c \cdot L_G) \cdot T$

Letting $E[A_D]$ be the expected limited loss gives the following entry ratios: $r_H = \frac{L_H}{E[A_D]}$ and $r_G = \frac{L_G}{E[A_D]}$

From this, we can draw the following Lee diagram



The green and yellow area is the **average ratable loss**, so $\frac{E[L]}{E[A_D]} = U + V$ [2]

Since the plan pays out on all losses, E[A], and we need to cover expenses and taxes we have $R = (e + E[A]) \cdot T$
Taking the expectation of the retrospective rating formula, [1], and equating gives
$$(e + E[A]) \cdot T = (B + c \cdot (U + V) \cdot E[A_D]) \cdot T \quad [3]$$

where we used [2] to replace E[L]

At the minimum premium, H, the area of U is zero so we get
$$H = (B + cL_H) \cdot T = (B + cVE[A_D]) \cdot T$$

which rearranges to $\frac{H}{T} - B = cVE[A_D]$ [4]

Substituting [4] into [3] yields $(e + E[A]) \cdot T = cUE[A_D] \cdot T + H$ [5]

However, from the Lee diagram we know $U = \phi_D(r_H) - \phi_D(r_G)$ [6]

Substituting [6] into [5] yields
$$(e + E[A]) \cdot T = c \cdot \left(\phi_D(r_H) - \phi_D(r_G)\right) \cdot E[A_D] \cdot T + H$$

which gives the first balance equation: $\boldsymbol{\phi_D(r_H) - \phi_D(r_G) = \frac{(e + E[A]) \cdot T - H}{c \cdot E[A_D] \cdot T}}$

Next, ratable losses associated with the minimum premium may be expressed as $L_H = r_H \cdot E[A_D]$
Applying the retrospective rating formula yields $H = (B + c \cdot r_H \cdot E[A_D]) \cdot T$
Similarly we have $G = (B + c \cdot r_G \cdot E[A_D]) \cdot T$

Taking the difference of these two equations gives $G - H = c \cdot (r_G - r_H) \cdot E[A_D] \cdot T$

which rearranges to the second balance equation: $\boldsymbol{r_G - r_H = \frac{G - H}{c \cdot E[A_D] \cdot T}}$

Note:
The first balance equation tells us $\phi_D(r_H) - \phi_D(r_G)$
(the green shaded area labelled by U) is the difference between the expected retrospective premium at the minimum ratable loss and the minimum premium as R = (e + E[A])T, scaled by the factor $cE[A_D]T$.