Reading: Mahler.Credibility Q1a_2018 (Problem 1)
Model: 2018.Q1a
Problem Type: Apply Chi-squared testing

Given An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
Accident Years	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	12,033,000	1,766,000	2,329,000	1,236,000	170	170
1996-2000	13,812,000	1,819,000	2,762,000	1,273,000	210	172
2001-2005	13,985,000	1,751,000	2,797,000	1,506,000	210	201
2006-2010	16,444,000	1,710,000	3,288,000	1,471,000	240	195
2011-2015	17,507,000	1,673,000	3,350,000	1,439,000	250	198
Total	73,781,000	8,719,000	14,526,000	6,925,000	1,080	936

Find

a) Conduct chi-squared tests with an α value of 0.10 on actual vs. expected claims counts to confirm whether or not risk parameters have shifted over time.

Use the following table of critical values:

Degrees of Freedom	Critical Value (α = 0.10)
1	2.706
2	4.605
3	6.251
4	7.779
5	9.236
6	10.645

Solution

a) This is part of an integrative question (IQ). As such, it's worth looking harder for potential wrinkles.
Looking at the ultimate claim counts it is clear both lines of business have experienced an increase in claims.
However, looking at the earned premiums, it's clear that LOB 1 has grown significantly more than LOB 2.
Hence, we'll need to **account for premium growth**.

We also notice that each group of accident years is the same size which makes it easier to calculate averages across years if needed.

We'll account for the growth in premiums by dividing the ultimate claim counts by earned premium. We get the following table:

Accident Years	LOB 1	LOB 2
1991-1995	0.00001413	0.00009626
1996-2000	0.00001520	0.00009456
2001-2005	0.00001502	0.00011479
2006-2010	0.00001459	0.00011404
2011-2015	0.00001428	0.00011835
Total	0.00001464	0.00010735

We'll use the Total row as the long-term average for each line of business.
Compute the expected claim counts for each group of accident years by multiplying the total claims per $ premium for the LOB by the earned premium for the group of accident years.

Accident Year	Expected Ultimate Claim Counts	
	LOB 1	LOB 2
1991-1995	176.1	189.6
1996-2000	202.2	195.3
2001-2005	204.7	188.0
2006-2010	240.7	183.6
2011-2015	256.3	179.6
Total	1,080	936

Note that if you round the expected ultimate claim counts to the nearest integer, you'll get a slightly different chi-squared statistic to the CAS answer.

The Chi-Squared statistic is $\sum \frac{(A-E)^2}{E}$

So we have the following:

LOB 1 Chi-squared = (170 - 176.1)^2 / 176.1 + (210 - 202.2)^2 / 202.2 + (210 - 204.7)^2 / 204.7 + (240 - 240.7)^2 / 240.7 + (250 - 256.3)^2 / 256.3
= 0.8063

LOB 2 Chi-squared = (170 - 189.6)^2 / 189.6 + (172 - 195.3)^2 / 195.3 + (201 - 188)^2 / 188 + (195 - 183.6)^2 / 183.6 + (198 - 179.6)^2 / 179.6
= 8.2978

We now need the degrees of freedom. Each line of business has five sets of accident years and we estimated a single average for each.
This gives 5 -1 = 4 degrees of freedom.

From the table given for $\alpha = 0.1$ at 4 degrees of freedom the critical value is 7.779

We accept the null hypothesis for line of business 1. That is, we cannot conclude LOB 1 has shifting risk parameters.

However, for LOB 2, we reject the null hypothesis and conclude the risk parameter is changing over time.