Reading: Mahler.Credibility
Model: 2018.Q1b
Problem Type: Apply Mean-Squared-Error criterion

Given

An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

Accident Years	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	12,033,000	1,766,000	2,329,000	1,236,000	170	170
1996-2000	13,812,000	1,819,000	2,762,000	1,273,000	210	172
2001-2005	13,985,000	1,751,000	2,797,000	1,506,000	210	201
2006-2010	16,444,000	1,710,000	3,288,000	1,471,000	240	195
2011-2015	17,507,000	1,673,000	3,350,000	1,439,000	250	198
Total	73,781,000	8,719,000	14,526,000	6,925,000	1,080	936

To select an expected future claim frequency for LOB 2, the actuarial consultant has decided to assign equal weight (Z/2) to each of the most recent two groups of accident years and the remaining weight (1-Z) to the overall mean frequency.

Find

b.) Calculate the expected future claim frequency per $1,000 premium for LOB 2 by first using the mean-squared-error (MSE) criterion to determine the optimal value for Z from the following three choices:

Z value	MSE
0.1	Not Provided
0.5	0.0190%
0.9	0.0164%

Solution

b.) We've added in "per $1,000 premium" into the CAS original exam question because without this, you could draw the wrong conclusion when comparing against the given MSE values. The mean-squared-error criterion means we'll select the value of Z which produces the lowest mean squared error. Consequently, we need to calculate the mean squared error when Z = 0.1.

Again, this is part of an Integrative Question, so we should be wary of additional wrinkles. In this case we'll need to account for the shift in earned premium and understand how to apply the credibility formula as well as calculate the mean squared error. All this before we can project the expected ultimate claims.
The question tells us to calculate the expected claims we need the overall average plus the two most recent groups of accident years.
This means we can calculate the expected claims for all bar the oldest and second oldest groups of accident years given as those lack data.

Further, since we're calculating the future expected claims at different points in time, we can't just use the Total row for our overall mean frequency because it may contain some years which haven't happened yet. As such, the average over "the whole period" will be a running average over all of the historical periods to date. Remember we're using Z = 0.1

Accident Years	Actual Ultimate Claims per $1,000 EP	Avg Ult Claims to date per $1,000 EP	Expected Ult Claims per $1,000 EP
1991-1995	0.0963	0.0963	NA (No prior information)
1996-2000	0.0946	0.0954	NA (Only 1 prior year of data)
2001-2005	0.1148	0.1018	0.0954
2006-2010	0.1140	0.1047	0.1021
2011-2015	0.1184	0.1074	0.1057

The CAS question wasn't very clear that the mean squared error values were also "per $1,000 of earned premium". If you converted the expected claims by multiplying by the earned premium, or if you used per $1million of premium, you get an answer that's a different order of magnitude. If this happens in an exam, it's a great clue to check your work or remember the question as potentially faulty. In the examiner's report the CAS was clear they accepted multiple answers in this situation as long as you correctly interpreted your result in the context of the given MSE values.

Now we have the expected ultimate claims per $1,000 we can apply the mean squared error formula: $mse = \sum \frac{(A-E)^2}{3}$

This formula is very close to the Chi-squared test formula. Instead of dividing by the expected value, E, we divide by the number of items we have data for to produce an average. In this case we have 3 groups of accident years.

mse = ((0.1148 - 0.0954)^2 + (0.114 - 0.1021)^2 + (0.1184 - 0.1057)^2) / 3
= 0.00022642
= 0.0226%

Comparing this result against the table given shows the lowest mse occurs at Z = 0.9

Now we've chosen the Z value, we can use all of the information available to finally answer the question and predict the future expected claim frequency. This would presumably be for accident years 2016-2020.

predicted = (0.9 / 2) * 0.1184 + (0.9 / 2) * 0.114 + (1 - 0.9) * 0.1074
= 0.1153

(Remember this is per $1,000 of earned premium).