Reading: Couret.Venter

Q5_2012 (Problem 1)

Model: 2012.Q5

Problem Type: Multi-Dimensional Credibility

Given The following data is used to price an excess of loss workers compensation policy:

- Data is available for the following injury types: Fatal, Permanent Total injury (PT), Major permanent partial (Major), minor permanent partial (Minor), temporary total (TT), and medical-only (Med).
- A multi-dimensional credibility technique (predicted) was used to estimate the frequency for class 5160.
- Class 5160 is in hazard group F.

Hazard Group F						
	Fatal	PT	Major	Minor	TT	Med
Frequency Relativity to TT	0.006	0.006	0.085	0.37	1.00	3.6
Severity Relativity to TT	80	100	30	4	1.00	0.3
Loss Elimination Ratio at $250,000	27%	22%	57%	100%	100%	100%

TT Frequency per $100 payroll	0.0002
TT Severity for Hazard Group F	$10,000

Hazard Group F for Fatal Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.75	0.70	0.90
Quintile 2	0.90	0.90	0.95
Quintile 3	1.00	1.00	1.00
Quintile 4	1.10	1.10	1.05
Quintile 5	1.25	1.30	1.10
Mean	1.00	1.00	1.00

Hazard Group F for PT Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.70	0.75	0.80
Quintile 2	0.90	0.90	0.90
Quintile 3	1.00	1.00	1.00
Quintile 4	1.15	1.20	1.10
Quintile 5	1.20	1.25	1.20
Mean	1.00	1.00	1.00

- The hazard group relativities for Major, Minor, TT, and Med will be used.
- The multi-dimensional credibility relativities for PT claims will be used.
- Class 5160 is in Quintile 4 for both Fatal and PT claims.

Find

a.) Determine whether multi-dimensional credibility relativities should be used to estimate the expected loss for fatal claims.

b.) Based on part a. above, calculate the expected loss for an excess of $250,000 workers compensation policy with $10 million in payroll.

Solution

This is a huge amount of information but the questions aren't actually that bad.

a.) We need to calculate the sum of squared errors for each of the credibility technique and the hazard group technique.
The data is already split into quintiles for us, and the overall means are 1.00 so no normalization is required.
We're told we only need to worry about the Fatal claims.

Multi-Dimensional Credibility Technique SSE (Predicted to Holdout)

SSE = (0.75 - 0.9)^2 + (0.9 - 0.95)^2 + (1 - 1)^2 + (1.1 - 1.05)^2 + (1.25 - 1.1)^2
= 0.050

Hazard Group SSE (Hazard group to Holdout)

SSE = (1 - 0.9)^2 + (1 - 0.95)^2 + (1 - 1)^2 + (1 - 1.05)^2 + (1 - 1.1)^2
= 0.025

Since the sum of squared errors is lower for the hazard group method, we won't use the multi-dimensional credibility relativities for fatal claims.

Note that for the hazard group, we used the mean of the entire hazard group **after normalization** which is 1.

b.) This part of the question is more like an IQ type of question. It blends knowledge from several papers.
From the first table we know all Minor, TT and Med losses are eliminated with a $250,000 attachment point.
This means we only need to look at Fatal, PT, and Major claims.

By our answer to part a. we'll use the fatal hazard group relativities, and in the question we're told to use the Major hazard group relativities.
A claim can't be both Fatal and Major so we calculate these separately. For PT claims we're told to use the credibility method relativities.
A key part of the problem is understanding how to relate the hazard group relativities to those produced by the credibility method or raw method.

To use the multi-dimensional credibilities or the raw credibilities, multiply the hazard group relativities by the quintile relativity which contains the class being priced.
If you're only using the hazard group relativities then you just multiply the hazard group relativities by 1.

Fatal relative to TT

Pure premium relativity =	0.48	(Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER =	0.3504	(Frequency x Severity) x (1 - LER)
Relative to hazard group =	0.3504	(Frequency x Severity) x (1 - LER) x 1.000

Major relative to TT

Pure premium relativity =	2.55	(Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER =	1.0965	(Frequency x Severity) x (1 - LER)
Relative to hazard group =	1.0965	(Frequency x Severity) x (1 - LER) x 1.000

PT relative to TT

This is more complicated since we're told to use the multi-dimensional credibility relativities.
We're told Class 5160 is in Quintile 4.
Using the Hazard Group F for PT Claims table, we look up the predicted value for quintile 4 which is 1.15.
This is the multi-dimensional credibility relativity for the class.

Pure premium relativity =	0.60	(Frequency x Severity) for hazard group before Loss Elimination Ratio
After LER =	0.468	(Frequency x Severity) x (1 - LER)
Credibility Relativity =	1.15	
Relative to hazard group =	0.5382	(Frequency x Severity) x (1 - LER) x 1.15

Summing the results gives the overall relativity to TT =	1.9851	
TT pure premium per $100 of payroll =	$2	By multiplying the frequency and severity in the second table.

We're told the company has $10 million in payroll. We convert this into $100s of payroll, multiply it by the TT pure premium and then multiply by the overall relativity to TT.

Answer to b.) $397,020